## Problems Based on Projectile Motion

**VERY SHORT ANSWER TYPE QUESTIONS :**

**Q.1** Give the vertical and horizontal component of acceleration of a body thrown horizontally with uniform speed.

**Sol.** The horizontal component of acceleration is zero while vertical component equals to the acceleration due to gravity.

**Q.2** Velocity of a projectile is $10ms^{-1}$ At what angle to the horizontal should it be projected so that it covers maximum horizontal distance.

**Sol.** 45º (The range of a projectile is maximum when it is projected at 45º with horizontal.

**Q.3** Give two example of projectile motion.

**Sol.** (a) Motion of a bullet fired from the gun.

(b) Motion of a javelin thrown by an athlete.

**Q.4** What is the angle of projection for maximum horizontal range?

**Sol.** 45º

**Q.5** Which component of velocity of a projective remains same throughout the time of flight.

**Sol.** Horizontal component.

**Q.6** A hunter aims his gun and fires directly at a monkey sitting on a tree. The moment the bullet leaves the barrel of the gun, the monkey falls.. Will the bullet hit the monkey?

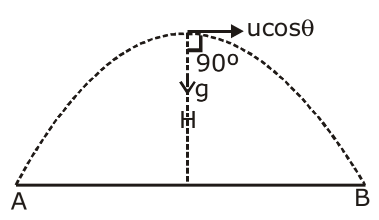
**Sol.** Yes, the bullet will hit the monkey. It is due to the fact that initial velocity of bullet and monkey in vertical direction is zero, so they fall through the same height.

**Q.7** Write an expression for the time of fight of a projectile fired at an angle $\theta$ with the vertical.

**Sol.** $T = \dfrac{2u\cos\theta}{g}$

**Q.8** What is the angle between the direction of velocity and acceleration at the highest point of a projectile path?

**Sol.** 90º




**Q.9** Why does the direction of motion of a projectile become horizontal at the highest point of its trajectory?

**Sol.** The direction of motion of a projectile becomes horizontal at the highest point of its trajectory because at highest point the vertical component of velocity is zero.

## Problems Based on Projectile Motion

**Q.10** A bomb is released from a horizontal flying plane when it is vertically above the target, shall it hit the target?

**Sol.** No.

**Q.11** (i) At which point of the projectile path the speed is minimum?

(ii) At which point the speed is maximum.

**Sol.** (i) At the highest point.

(ii) At the point of projection.

**Q.12** A hunter aims his gun at a bird, sitting on a tree and fires. Will the bullet hit the bird?

**Sol.** No, the bullet will not high the bird. the bullet falls downward under gravity and will pass, a small distance below the bird.

**Q.13** A football is thrown in a parabolic path. Is there any point at which the acceleration is perpendicular to the velocity?

**Sol.** At the highest point.

**Q.14** A bob of mass 0.1kg hung from the ceiling of a room by a string 2m long is set into oscillation. The speed of the bob at its mean position is $1ms^{-1}$. What is the trajectory of the bob if the string is cut when the bob is (a) at one of its extreme positions, (b) at its mean position.

**Sol.** (a) Straight path (b) Parabolic path.

**Q.15** A body is projected so that it has maximum range R. What is the maximum height reached during the flight?

**Sol.** $\frac{R}{4}$

**Q.16** A bullet P is fired from a gun when the angle of elevation of the gun is 30º. Another bullet Q is fired from the gun when the angle of elevation is 60º. Which of the of bullets would have a greater horizontal range?

**Sol.** Both will have the same horizontal range.

**Q.17** A projectile is projected at an angle of 30º to the horizontal with sped v. Another projectile of double the mass is projected with speed v at some other angle $\theta$. What is $\theta$ if the horizontal range is same in both the cases?

**Sol.** Mass has no role to play

$\theta = 90º - 30º = 60º$

**Q.18** A particle moves in a plane with uniform acceleration in a direction different from its initial velocity. What path the particle shall follow?

**Sol.** Parabolic path.

## Problems Based on Projectile Motion

**Q.19** A body is projected at an angle of 60º with the horizontal with momentum p. What is the magnitude of its momentum at its highest point?

**Sol.** $p\cos 60^\circ = \frac{p}{2}$

**Q.20** A ball of mass m is thrown vertically upwards. another ball of mass 2m is thrown up making an angle $\theta$ with the vertical. Both of them stay in air for the same time. What is the ratio of their maximum heights?

**Sol.** Since the two bodies are in air for equal intervals of time therefore the velocity of projection of first body is equal to the vertical component of the velocity of projection of the second body. So, the maximum heights are the same. The required ratio is 1 : 1.

**Q.21** A ball is thrown at an angle of 45º to the horizontal with kinetic energy K. What is the kinetic energy at the highest point of the trajectory?

**Sol.** Kinetic energy at the highest point = $mv^2 \cos^2 45^\circ = \frac{K}{2}$

**Q.22** State whether the following statements are true or false :

(i) A projectile fired from the ground follows a parabolic path. The speed of the projectile is minimum at the top of the path

(ii) Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed (neglect air resistance).

**Sol.** (i) true (ii) true

**Q.23** Is it possible to have two dimensional motion with an acceleration in only one dimension?

**Sol.** Yes. The motion of the projectile is a two dimensional motion. But the acceleration is in only on dimension.

**Q.24** Can a body have zero velocity and still be accelerating?

**Sol.** Yes. When a body is at the highest point of its motion, its velocity is zero but its acceleration is equal to the acceleration due to gravity.

**SHORT ANSWER TYPE - I QUESTIONS :**

**Q.25** Two bombs of 2kg and 3kg are thrown from a can with the same velocity in the same direction, which bomb will reach first?

**Sol.** Both the bodies will reach simultaneously as time of flight does not depend on mass of body.

**Q.26** What is the ratio of maximum horizontal range to the maximum height attained by a projectile?

**Sol.** Maximum horizontal range = $\frac{u^2}{g}$

Maximum height $= \frac{u^2 \sin^2\theta}{2g}$

$\therefore \frac{\text{Maximum horizontal range}}{\text{Maximum height}} = \frac{2}{\sin^2\theta}$

**Q.27** Two particles are moving with equal and opposite velocities in such a way that they are always at a constant distance d apart. Calculate the time after which the particles return to their initial positions.

**Sol.** Clearly, the two particles are at the two ends of the diameter of a circular path. It is further clear that each particle will return to its initial position after describing one circle.

Required time = $\frac{2\pi r}{v} = \frac{\pi d}{v}$, where v represents the speed.

**Q.28** A horizontal stream of water leaves an opening in the side of a tank. If the opening is h metre above the ground and the stream hits the ground D metre away, then what is the speed of water as it leaves the tank in terms of g, h and D?

**Sol.** The given problem is the problem of horizontal projectile. The stream of water shall follow the parabolic path.

Now, $t = \sqrt{\frac{2h}{g}};\ v = \frac{D}{t} = D\sqrt{\frac{g}{2h}}$

**SHORT ANSWER TYPE - II QUESTIONS :**

**Q.29** Prove that max horizontal range is four times the maximum height attained by the projectile, when fired at an indication so as to have maximum horizontal range.

**Sol.** The maximum horizontal range is given by

$$R_{max} = \frac{u^2}{g} \quad (\text{when } \theta = 45^\circ)$$

The maximum height attained

$$H = \frac{u^2 \sin^2\theta}{2g}$$

When $\theta = 45^\circ$ $H = \frac{u^2}{2g}(\sin 45^\circ)^2$

$$= \frac{u^2}{4g}$$

$H = \frac{1}{4}R_{max}$ Using equation (1)

$\therefore \quad R_{max} = 4H$

## Problems Based on Projectile Motion

**Q.30** The range of a projectile fired at 30º with horizontal is R. If the angle projection is increased to 60º what will be the value of new range.

**Sol.** The range of a projectile is given by

$$R = \frac{u^2}{g}\sin 2\theta$$

When $\theta = 30º$ $R = \frac{u^2}{g}\sin 2 \times 30º$

$$R = \frac{u^2}{g} \times \left(\frac{\sqrt{3}}{2}\right)$$

When $\theta = 60º$ $R = \frac{u^2}{g}\sin 2 \times 60º$

$$= \frac{u^2}{g}\left(\frac{\sqrt{3}}{2}\right)$$

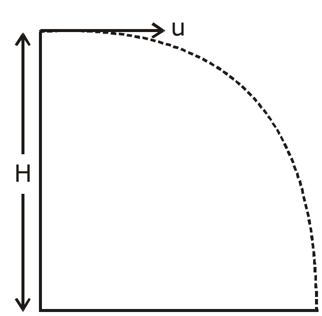
The range will remain same i.e. R.

**Q.31** A body is thrown horizontally with a velocity v from a lower H meter high. After how much time and at what distance from the base of the tower will be body strike the ground?

**Sol.** Let time taken is t

$$h = ut + \frac{1}{2}gt^2$$

$$H = 0 + \frac{1}{2}$$




or $t = \sqrt{\frac{2H}{g}}$

Also range = Horizontal velocity × time of flight

$$= v \times \sqrt{\frac{2H}{g}}$$

$$= v\sqrt{\frac{2H}{g}}$$

## Problems Based on Projectile Motion

**Q.32** Two bodies are thrown with the same velocity at angle $\alpha$ and $(90 - \alpha)$ with the horizontal. What will be the ratio of the horizontal ranges.

**Sol.** The range in the two cases will remain same.

$$R_1 = \frac{u^2}{g}\sin 2\alpha$$

When angle of projection is $(90 - \alpha)$ then

$$R_2 = \frac{u^2}{g}\sin 2(90 - \alpha)$$

$$= \frac{u^2}{g}\sin(180 - 2\alpha)$$

$$R_2 = \frac{u^2}{g}\sin 2\alpha$$

$\therefore$ $R_1 : R_2 = 1 : 1$ i.e. both have same range.

**Q.33** Derive an expression of maximum horizontal range.

**Sol.** The horizontal displacement of a the projectile during the time for flight called horizontal range.

i.e. Horizontal range = Horizontal velocity X time of flight.

$$= u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$= 2\frac{u^2}{g}\sin\theta\ \cos\theta$$

$$= \frac{u^2}{g}(2\sin\theta\ \cos\theta)$$

$$= \frac{u^2}{g}\sin\theta$$

For maximum range $\sin 2\theta = 1$

$$\therefore \quad R_{max} = \frac{u^2}{g}$$

## Problems Based on Projectile Motion

**Q.34** Derive an expression for velocity of a projectile at any instant of time projected with velocity u inclined at an angle $\theta$ with horizontal.

**Sol.** The horizontal component of velocity after time t remains same is

$V_x = u\cos\theta$

Let vertical component of velocity after time t is $V_y$.

$\therefore \quad V_y = u\sin\theta - gt \quad (v = u + gt)$

As x and y component of velocity are mutually perpendicularly to each other the resultant velocity is given by

$$V = \sqrt{v_x^2 + V_y^2}$$

$$= \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

$$= \sqrt{u^2\cos^2\theta + u^2\sin^2\theta + g^2t^2 - 2ugt\sin\theta}$$

$$= \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

**Q.35** Prove that a projectile covers the same horizontal range for two angles of projection.

**Sol.** The horizontal range of a projectile, projected with velocity u making an angle $\theta$ with the horizontal direction is given by

$$R_1 = \frac{u^2}{g}\sin 2\theta \qquad ....(1)$$

When the same projectile is projected at an angle $(90 - \theta)$, with the horizontal direction then horizontal range will be

$$R_2 = \frac{u^2}{g}\sin 2(90 - \theta)$$

$$= \frac{u^2}{g}\sin(180 - 2\theta)$$

$$= \frac{u^2}{g}\sin 2\theta$$

From (1) and (2) we have

$R_1 = R_2$

i.e. the horizontal range is the same whether the angle of projection of projectile is $\theta$ or $(90 - \theta)$

## Problems Based on Projectile Motion

**Q.36** Show that the projection angle $\theta_0$ for a projectile launched from the origin is given by.

$$\theta = \tan^{-1}\left(\frac{4h_m}{R}\right),$$ where the symbols have their usual meanings.

**Sol.** Maximum height, $h_m = \frac{v^2 \sin^2\theta_0}{2g}$

Horizontal range, $R = \frac{v^2 \sin 2\theta_0}{g}$

Now, $$\frac{h_m}{R} = \frac{v^2\sin^2\theta_0}{2g} \times \frac{g}{v^2 \sin 2\theta_0} = \frac{\sin^2\theta_0}{2 \times 2\sin\theta_0\cos\theta_0} = \frac{\tan\theta_0}{4}$$

or $$\tan\theta_0 = \frac{4h_m}{R}$$

or $$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

**Q.37** Two balls are thrown with the same initial velocity at angle $\alpha$ and $(90 - \alpha)$ with the horizontal. What will be the ratio of the maximum heights attained by them?

**Sol.** $h_1 = \frac{v^2\sin^2\alpha}{2g}$ ;

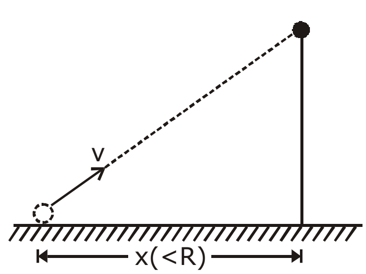
$$h_2 = \frac{v^2\sin^2(90-\alpha)}{2g} = \frac{v^2\cos^2\alpha}{2g}$$

$$\frac{h_1}{h_2} = \frac{v^2\sin^2\alpha}{2g} \times \frac{2g}{v^2\cos^2\alpha} = \frac{\tan^2\alpha}{1}$$

**LONG ANSWER TYPE QUESTIONS :**

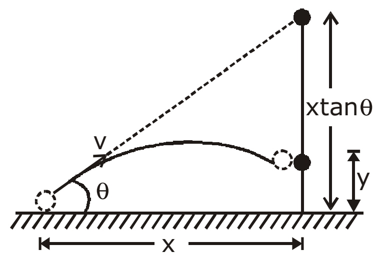
**Q.38** A ball (shown by dotted circle) is projected directly towards a second ball (shown by dark circle. The horizontal distance x for th second ball is less than the horizontal range R of the first ball. The second ball is released from rest at the instant the first is projected. Will the two balls collide?

**Sol.** Yes. Initial elevation of second ball = $x\tan\theta$.



During time t, the second ball covers a distance = $\frac{1}{2}gt^2$.

If y is the elevation at the instant of collision, then




$$y = x\tan\theta - \frac{1}{2}gt^2 \quad ........(1)$$

For the second ball,

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

Also, $x = v\cos\theta \times t$

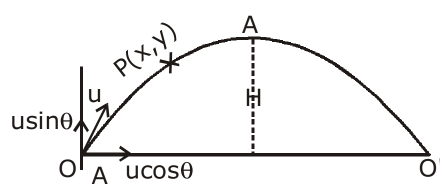
or $t = \frac{x}{v\cos\theta}$

$\therefore$ $y = x\tan\theta - \frac{1}{2}gt^2 \quad ........(2)$

If follows from (1) and (2) that both the balls again the same elevation at time t. So, the two must collide, irrespective of the initial velocity of the first ball.

**Q.39** Prove that a projectile fired with a velocity u making an angle θ with the horizontal follows a parabolic path.

**Sol.** Consider a projectile if fired with velocity u making an angle θ with the horizontal, from the point O.




The velocity u may be resolved into two components, u cos θ is horizontal component and u sin θ the vertical component. If we neglect friction duty to air, then horizontal component u cos θ will remain constant. The position of the particle (projectile) along x axis after time t is given by

$x = u\cos\theta \times t$

or $t = \frac{x}{u\cos\theta}$ ......(1)

The position of the projectile along y axis after time t is given by

$y = (u\sin\theta)\, t - \frac{1}{2}gt^2$ .......(2)

Substituting the value of r from equation (1) in equation (2)

$$y = u\ \sin\theta \left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x^2}{u^2\cos^2\theta}\right)$$

$y = x\ \tan\theta - \frac{g}{2u^2\ \cos^2\ \theta}x^2$ .......(3)

Equation (3) is the equation of a parabola. Thus the trajectory of a projectile fired at an angle $\theta$ with the horizontal is a parabola.

**Q.40** A projectile is fired with a velocity u making an angle $\theta$ with the vertical. Obtain the expression for (a) maximum height (b) time of flight (c) horizontal range.

**Sol.** (i) At maximum height the vertical component u $\sin\theta$ become zero

Therefore $u = u\sin\theta$ $v = 0$

Let H is the maximum height attained by projectile

$v^2 = u^2 + 2gH$

$o = u^2\ \sin^2\theta - 2gH$

or $H = \frac{u^2\sin^2\theta}{2g}$

(ii) The time during which the projectile remains in air is called time flight.

The time taken by projectile to go from O to A equals to time taken to go from A to O'. Say this time is (T/2)

$\therefore$ $v = u + gt$

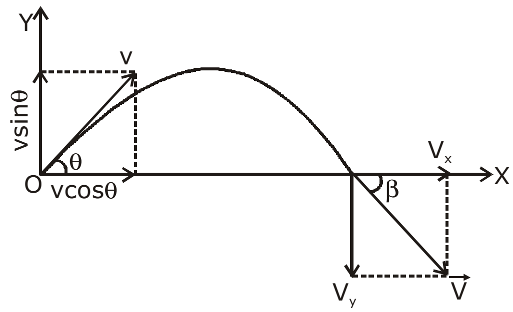
$o = u\sin\theta - g\ (T/2)$

or $T = \frac{2u\sin\theta}{g}$

## Problems Based on Projectile Motion

**Q.41** Prove that the velocity at the end of flight of an oblique projectile is the same in magnitude as the beginning but the angle that it makes with the horizontal is negative of the angle of projection.

**Sol.** Let $\vec{V}$ be the velocity of the projectile at the end of flight. Let $V_x$ and $V_y$ be the horizontal and vertical components respectively. Since horizontal motion is uniform motion.




$\therefore \quad V_x = v\cos\theta \quad$ Again, $\quad V_y = v\sin\theta - gT$

where T is the time of flight

$$V_y = v\sin\theta - g \times \frac{2v\sin\theta}{g}$$

or $\quad V_y = -v\sin\theta$

Now $\quad V = \sqrt{V_x^2 + V_y^2}$

or $\quad V = \sqrt{(v\cos\theta)^2 + (-v\sin\theta)^2} = \sqrt{v^2(\cos^2\theta + \sin^2\theta)} = v$

So, the magnitude of the velocity of the end of the flight is equal to the magnitude of the velocity of projection.

If $\beta$ is the angle which the velocity $\vec{V}$ makes with the horizontal, then

$$\tan\beta = \frac{V_y}{V_x} = \frac{-v\sin\theta}{v\cos\theta}$$

or $\quad \tan\beta = -\tan\theta$ or $\tan\beta = \tan(-\theta)$ or $\beta = -\theta$

So, the velocity vector at the end of flight makes an angle '$-\theta$' with the horizontal. This angle is negative of the angle of projection.

**NUMERICALS :**

**Q.42** A ball is thrown at an angle $\theta$ with the horizontal. Its time of flight is 2 second. Its horizontal range is 100metre. What is the horizontal component of the velocity of projection? Give : $g = 10ms^{-1}$.

**Sol.** $2 = \dfrac{2v\sin\theta}{10}$ or $v\sin\theta = 10ms^{-1}$

or $\quad 100 = \dfrac{2(10)v\cos\theta}{10} \quad v\cos\theta = 50ms^{-1}$

## Problems Based on Projectile Motion

**Q.43** A projectile of mass m is thrown with velocity v from the ground at an angle of 45º with the horizontal. What is the magnitude of change in momentum between leaving and arrival back at the ground?

**Sol.** Magnitude of change in momentum

$$= mg \times \frac{2v\sin\theta}{g} = 2mv\sin 45^\circ$$

$$= \sqrt{2}\,mv$$

**Q.44** What is the maximum vertical height to which a baseball player can throw a ball if he can throw it to a maximum horizontal distance of 100m?

**Sol.** $\frac{v^2}{g} = 100$; maximum vertical height

$$= \frac{v^2}{2g} = \frac{100}{2}m = 50m$$

**Q.45** A bomb is dropped from an aeroplane flying horizontally with a velocity of $720 kmh^{-1}$, at an altitude of 980m. After what time the bomb will hit her ground?

**Sol.** $$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 980}{9.8}}s = 10\sqrt{2} = 14.14\,s$$

**Q.46** A bullet fired at an angle of 30º with the horizontal hits the ground 3 km away. By adjusting it a angle of projection can one hope to hit a target 5km away? Assume the muzzle speed to be fixed and neglect air resistance.

**Sol.** Here $\theta = 30^\circ$

$R = 3km$

From formula $R = \frac{u^2}{g}\sin 2\theta$

$$3 = \frac{u^2}{g}\sin 2\theta \quad ....(1)$$

Let $R_{max}$ is the maximum range then

$$R_{max} = \frac{u^2}{g} \quad ....(2)$$

Dividing equation (2) by (1)

$$\frac{R_{max}}{3} = \frac{1}{\sin 60^\circ}$$

or $R_{max} = 3.46m$

As $R_{max} < 5km$, the bullet cannot hit the target.

## Problems Based on Projectile Motion

**Q.47** One of the rectangular components of a velocity $300kmh^{-1}$ is 50km/h. Find the other component.

**Sol.** Here $u = 200kmh^{-1}$

Let one component is $u_x = u\cos\theta = 50kmh^{-1}$

We know $u^2 = u_x^2 + u_y^2$

$\therefore$ $u_y^2 = u^2 - u_x^2$

$= 500^2 - 50^2 = 37500$

or $u_y = \sqrt{37500} = 193.6kmh^{-1}$

**Q.48** The ceiling of a long hall is 25m high. What is the maximum horizontal distance that a ball thrown with a speed of $40ms^{-1}$ can go without hitting the ceiling of the ball?

**Sol.** Here $u = 40ms^{-1}$

Height of ceiling (H) = 25m

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$25 = \frac{40^2 \sin^2\theta}{2 \times 10}$$

or $\sin\theta = \frac{\sqrt{5}}{4}$

Now $\sin^2\theta = \cos^2\theta = 1$

$$\cos^2\theta = 1 - \frac{5}{16} = \frac{11}{16}$$

$$\cos\theta = \frac{\sqrt{11}}{4}$$

Now range $R = \frac{u^2}{g}(2\sin\theta\ \cos\theta)$

$$= \frac{(40)^2}{10} \times 2\left(\frac{\sqrt{5}}{4}\right)\left(\frac{\sqrt{11}}{4}\right)$$

or $R = 151.3m$

**Q.49** Find the angle of projection at which the horizontal range and maximum height of a projectile are equal.

**Sol.** Horizontal range $R = \frac{u^2}{g}(2\sin\theta\ \cos\theta)$

Maximum height $= \frac{u^2 \sin^2\theta}{2g}$

$\therefore \quad \frac{u^2}{g}(2\sin\theta\cos\theta) = \frac{u^2\sin^2\theta}{2g}$

or $\quad \cos\theta = \frac{\sin\theta}{4}$

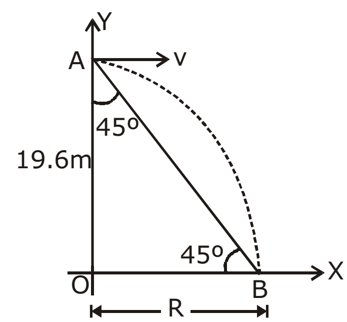
or $\quad \tan\theta = 4 = \tan 75.96^\circ$

$\theta = 75.96^\circ$

**Q.50** From the top of a building 19.6m high, a ball is projected horizontally. After how long does it strike the ground? If the line joining the point of projection to the point where it hits the ground makes an angle of 45º with the horizontal. What is the initial velocity of the ball.

**Sol.** Here initial vertical velocity = 0

$h = 19.6\text{m} \qquad g = 9.8\text{ms}^{-2}$




$h = ut + \frac{1}{2}gt^2$

$19.6 = 0 + \frac{1}{2} \times 9.8t^2$

or $\quad t = 2\text{sec}$

B is the point where the ball strikes

$\therefore \quad OA = OB = 19.6$

Let horizontal velocity is v

$\therefore \quad$ or $u \times t = 19.6$

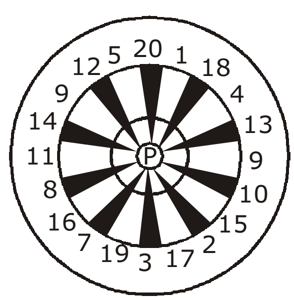
or $u = \frac{19.6}{t}$

$= \frac{19.6}{2}$

$= 9.8\text{ms}^{-1}$

**Q.51** A dart is thrown horizontally toward the bull's-eye, point P on the dart board of fig., with an initial speed of $10\text{ms}^{-1}$. It hits at point Q on the rim, vertically below P, 0.19s later. (a) What is the distance PQ? (b) How far away from the dart board did the dart thrown stand?

**Sol.** (a) Consider the vertical motion of the projectile.



Initial velocity in the vertical direction, $v_y(0) = 0$

$y(0) = 0,\ a_y = 9.8ms^{-2}$

Using $y(t) = y(0) + v_y(0)\,t + \frac{1}{2}a_y t^2$, we get

$$y = \frac{1}{2} \times 9.8 \times 0.19 \times 0.19m = 0.18m = 18cm$$

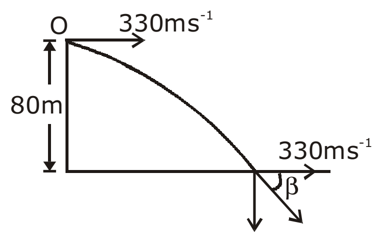
(b) Using $x = vt$, we get

$x = 10ms^{-1} \times 0.19s = 1.9s$

**Q.52** A projectile is fired with a horizontal velocity of $330ms^{-1}$ from the top of a cliff 80m high. How long will it take to strike the level ground at the base of the cliff? With what velocity will it strike? Neglect air resistance.

**Sol.** Let us choose the top of the cliff as the origin. Let the time 't' at which stone is thrown from origin O be taken as zero. Let the direction of initial velocity be taken as the positive direction of X-axis. Let the vertically upward direction be taken as the positive direction of Y-axis.

Let us consider the vertical motion.




We know that $y(t) = y(0) = v_y\,(0)\,t + \frac{1}{2}\,a_y t^2$

Here, $y(t) = -80m,\ y(0) = 0$

$v_y(0) = 0,\ a_y = -9.8ms^{-2}$

$\therefore \quad -80 = -\frac{1}{2} \times 9.8 \times t^2$

or $\quad t^2 = \frac{80}{4.9}$ or $t = 4.04s$

Again, $v_y(t) = v_y(0) + a_y t$

$v_y = 0 - 9.8 \times 4.04\ ms^{-1} = -39.59ms^{-1}$

The -ve sign indicates that the velocity is in the vertically downward direction.

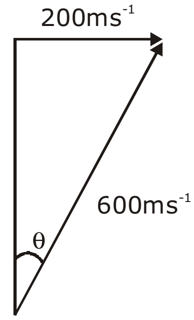
speed $= \sqrt{v_x^2 + v_y^2} = \sqrt{330^2 + (-39.59)^2}ms^{-2} = 332.37ms^{-1}$

The angle $\beta$ is given by $\tan\beta = \frac{39.59}{330} = 0.12$ or $\beta = 6.84^o$

**Q.53** A fighter plane flying horizontally at an altitude of 1.5km with speed $720 kmh^{-1}$ passes directly overhead an antiaircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed $600 ms^{-1}$ to hit the plane? At what minimum altitude should be pilot fly the plane to avoid being hit? Given : $g = 10 ms^{-2}$

**Sol.** Velocity of plane, $v_p = 720 \times \frac{5}{18} ms^{-1} = 200 ms^{-1}$

Velocity of shell = $600 ms^{-1}$ ; $\sin\theta = \frac{200}{600} = \frac{1}{3}$




or $\theta = \sin^{-1}\left(\frac{1}{3}\right) = 19.47^o$

Let h be the required minimum height.

Using $v^2 - u^2 = 2aS$, we get

$0^2 - (600 \cos\theta)^2 = -2 \times 10 \times h$

or $h = \frac{600 \times 600\,(1 - \sin^2\theta)}{20}$

$= 30 \times 600 \left(1 - \frac{1}{9}\right) = \frac{8}{9} \times 30 \times 600 m = 16 km$

**Q.54** A projectile is thrown with an initial velocity of $x\hat{i} + y\hat{j}$. The range of the projectile is twice the maximum height of the projectile. Calculate = $\frac{y}{x}$ .

**Sol.** $\frac{v^2 \sin 2\theta}{g} = 2\frac{v^2 \sin^2\theta}{2g}$

or $2v^2 \sin\theta \cos\theta = v^2 \sin^2\theta$

or $2(v\sin\theta)(v\cos\theta) = (v\sin\theta)(v\sin\theta)$

But $v\sin\theta = y$ and $v\cos\theta = x$ (given)

$\therefore$ $2yx = y^2$ or $2x = y$

or $\frac{y}{x} = 2$

**Q.55** A projectile is thrown at an angle θ with the horizontal with kinetic energy E. Calculate the potential energy at the topmost point of the trajectory.

**Sol.** Potential energy at the topmost point of the trajectory = $mgh_{max}$

$= mg\frac{v^2 \sin^2\theta}{2g} = \left(\frac{1}{2}mv^2\right)\sin^2\theta = E\sin^2\theta$

## Problems Based on Projectile Motion

**Q.56** A projectile has a range of 50m and reaches a maximum height of 10m. What is the elevation of the projectile?

**Sol.** We know that horizontal range, $R = \frac{v^2 \sin 2\theta}{g} = \frac{2v^2 \sin\theta \cos\theta}{g}$

Maximum height, $H = \frac{v^2 \sin^2\theta}{2g} \times \frac{g}{2v^2 \sin\theta \cos\theta}$

or $\frac{H}{R} = \frac{1}{4}\tan\theta$

or $\tan\theta = \frac{4H}{R}$

or $\tan\theta = \frac{4 \times 10}{50} = \frac{4}{5} = 0.8$ or $\theta = 38.66°$

**Q.57** A cricket ball is thrown at a speed of $28 ms^{-1}$ in a direction 30º above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level, and (c) the distance from the thrower to the point where the ball returns to the same level.

**Sol.** The maximum height is given by

$$h_{max} = \frac{(v\sin\theta)^2}{2g} = \frac{(28\sin 30°)^2}{2(9.8)} m = \frac{14 \times 14}{2 \times 9.8} = 10.0m$$

(b) The time taken to return to the same level is

$= (2v \sin\theta)/g = (2 \times 28 \times \sin 30°)/9.8 = 28/9.8s = 2.9s$

(c) The distance from the thrower to the point where the ball returns to the same level is

$$= \frac{v_0^2 \sin 2\theta_0}{g} = \frac{28 \times 28 \times \sin 60°}{9.8} = 69.3m$$

**Q.58.** A cricketer can throw a ball to a maximum horizontal distance of 01m. How much high above the ground can the cricket throw the same ball?

**Sol.** Maximum horizontal range = 100m

$\therefore$ $\frac{v^2}{g} = 100$ ....(1)

We know that $v(t)^2 - v(0)^2 = 2a[x(t) - x(0)]$

Now, $v(t) = 0$, $v(0) = v$, $x(t) - x(0) = h$

$\therefore$ $0^2 - v^2 = 2(-g)h$

or $h = \frac{1}{2} + \frac{v^2}{g}$ or $h = \frac{1}{2} \times 100m = 500m$ [from equation (1)]

## Problems Based on Projectile Motion

**Q.59** From the same point, two balls A and B are thrown simultaneously. A is thrown vertically up with a velocity of $20ms^{-1}$. B is thrown with a velocity of $20ms^{-1}$ at an angle of 60º with the vertical. Determine the separation between the two balls at t = 1 second.

**Sol.** Let the horizontal separation between the wall balls be denote by x.

Then x = 20sin60º × 1 = $20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}m$

Let the vertical separation the between the two balls be denoted by y.

Then y = $\left[20 \times 1 - \frac{1}{2}g \times 1^2\right] - \left[20\cos 60º \times 1 - \frac{1}{2}g \times 1^2\right]$

or y = 20 – 20cos60º = 20 – 10 = 10m

Separation between two balls = $\sqrt{x^2 + y^2} = \sqrt{(10\sqrt{3})^2 + (10)^2} = \sqrt{400}m = 20m$

**Q.60** A projectile is thrown with velocity v at an angle, θ with the horizontal. Its maximum height is 8m and horizontal range is 24m. Calculate v in terms of acceleration due to gravity g. Also determine sinθ.

**Sol.** $h_{max} = \frac{v^2 \sin^2\theta}{2g} = \frac{v_y^2}{2g};\ 8 = \frac{u_y^2}{2g}$

or $u_y^2 = 16g$ or $v_y = 4\sqrt{g}$

Again, $R = \frac{v^2 \sin 2\theta}{g} = \frac{2v^2 \sin\theta\cos\theta}{g}$

or $R = \frac{2v_y v_x}{g}$

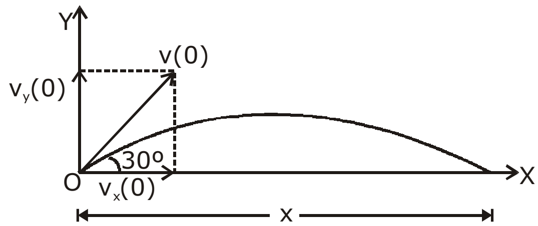
∴ $24 = \frac{2 \times 4\sqrt{g} \times v_x}{g}$ or $v_x = 3\sqrt{g}$

Now, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{9g + 16g} = \sqrt{25g} = 5\sqrt{g}$

Now, $\sin\theta = \frac{v_y}{v} = \frac{4\sqrt{g}}{5\sqrt{g}} = 0.8$

**Q.61** A ball is thrown with an initial velocity of $100ms^{-1}$ at an angle of 30º above the horizontal. How far from the throwing point will the ball attain its original level? Solve the problem without using formula for horizontal range.

**Sol.** Let the point O from where the ball is thrown be chosen as the origin (x = 0 and y = 0). Let the time 't' when the projectile is thrown be taken as zero.




The initial velocity v(0) of the projectile may be resolved into two rectangular components $v_x(0)$ and $v_y$ (0) = v(0) cos30º

= 100 × 0.866 $ms^{-1}$ = 86.6$ms^{-1}$

and $v_y$ (0) = v(0) sin30º = 100 × 0.5$ms^{-1}$ = 50$ms^{-1}$

The motion of the projectile can be divided into two parts horizontal motion and vertical motion. The horizontal motion is uniform motion while the vertical motion is an accelerated motion. Let us consider the vertical motion.

$$y(t) = y(0) = v_y(0)t + \frac{1}{2}a_y t^2$$

Now, y(t) = 0 [∵ The ball returns to the original level.]

$v_y$ (0) = 50$ms^{-1}$

$a_y$ = –g = –9.8$ms^{-2}$

y(0) = 0

$$\therefore \quad 0 = 0 + 50t + \frac{1}{2}(-9.8)t^2$$

or $4.9t^2 - 50t = 0$ or $t(4.9 - 50) = 0$

or t = 0 and 4.9t – 50 = 0

The value t = 0 represents the initial situation.

$$\therefore \quad 4.9t - 50 = 0 \quad \text{and} \quad t = \frac{50}{4.9}s = 10.2s$$

Let us no consider the horizontal motion

$x(t) = v_x(0)t = 86.6 \times 10.2m = 883.32m$

**Q.62** A particle starts from the origin at t = 0s with a velocity of $10.0\hat{j}$ $ms^{-1}$ and moves in the x-y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})ms^{-2}$. (a) At what time is the x-coordinate of the particle 16m? What is the y-coordinate of the particle at that time? (b) What is the speed of the particle at that time?

**Sol.** $\vec{u} = 10.0\hat{j},\ \vec{a} = 8.0\hat{j} + 2.0\hat{j}$

$$\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{r} = 10.0t\vec{j} + \frac{1}{2}(8.0\vec{i} + 2.9\vec{j})t^2$$

$$\vec{r} = 4.0t^2\hat{i} + (10.0t + 1.0t^2)\hat{j}$$

(a) x co-ordiante $= 4.0t^2\hat{i} = 16$ or t = 2s

y co-ordiante = 10.0 × 2 + 1.0 × 2 × 2 = 24m

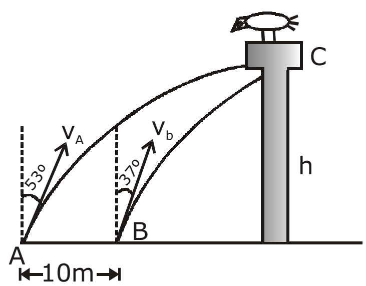
(b) $\vec{v} = \frac{d}{dt}(\vec{r}) = 8t\hat{i} + (10.0 + 2t)\hat{j}$

At t = 2, $\vec{v} = 16\vec{i} + 14\vec{j}$

$$v = \sqrt{16^2 + 14^2} = \sqrt{256 + 196} = \sqrt{452} = 21.26 \text{ ms}^{-1}$$

**Q.63** Two boys standing at A and B fire bullets simultaneously at a bird stationary at C. The bullets are fired from A and B at angles of 53º and 37º respectively with the vertical. Both the bullets hit the bird simultaneously. What is the value of $v_A$ if $v_B$ = 60 units? Given : tan37° = $\frac{3}{4}$ and AB = 10m.

**Sol.** The vertical components must be equal.




∴ $v_A\cos53º = v_B = \cos37º$

or $v_A = v_B \frac{\cos37º}{\cos(90º - 37°)}$

or $v_A\ 60\cos37º = \frac{60}{\tan37º}$

$$= \frac{64 \times 4}{3} = 80\text{units.}$$