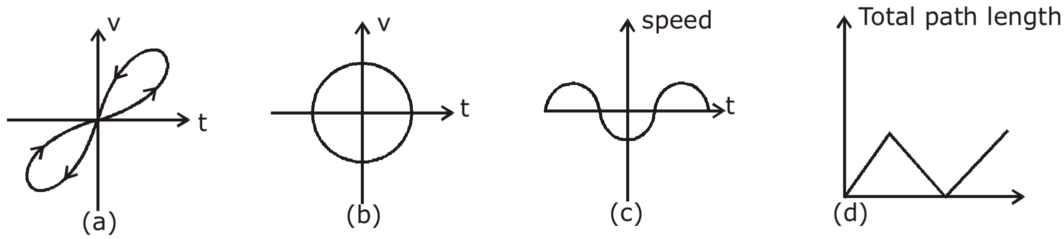
SHORT ANSWER TYPE QUESTIONS :

Q.1 Look at the graphs (a) to (d) carefully and state with reasons, which of these cannot possibly represent one dimensional motion of a particle.



Sol. (a) If we draw a line parallel to y-axis, it cuts the graph at two points. It shows two positions of the objects at same time which is not possible.

(b) This graph also shows two velocities of an objects for the same value of time, which is not possible.

(c) This graph shows that speed is negative for certain time, which is not possible.

(d) This graph is also impossible as the total path covered by a body can never by zero.

Q.2 Two particles located at a point begin to move with velocity is $4ms^{-1}$ and $1ms^{-1}$ horizontally in opposite directions. Determine the time when their velocity vectors perpendicular. Assume that the motion takes place in a uniform gravitational field of strength g.

Sol. Velocity of first particle at time $t = 4\hat{i} - gt\hat{j}$

Velocity of second particle at time $t = -\hat{i} - gt\hat{j}$

Since the dot product of perpendicular vectors is zero therefore $(4\hat{i} - gt\hat{j})\,(-\hat{i} - gt\hat{j}) = 0$

or $-4 + g^2t^2 = 0$

or $g^2t^2 = 4$

or $t = \dfrac{2}{g}$

LONG ANSWER TYPE QUESTIONS :

Q.3 A car accelerates from rest a constant rate α for some time. After which it decelerates at a constant rate β to come to rest. If the total time taken is t seconds evaluate (i) the maximum velocity reached (ii) the total distance travelled.

Sol. Let for time t_1 the car accelerates and for next time t_2 the car accelerates so that

$t = t_1 + t_2$

v is the max. velocity

$\therefore \quad \alpha = \dfrac{v}{t_1} \quad$ or $\quad t_1 = \dfrac{v}{\alpha}$

$\beta = \dfrac{v}{t_2} \quad$ or $\quad t_2 = \dfrac{v}{\beta}$

$\therefore \quad t = t_1 + t_2 = \frac{v}{\alpha} + \frac{v}{\beta}$

or $\quad t = v \left[\frac{1}{\alpha} + \frac{1}{\beta}\right]$

$= v\left[\frac{\alpha+\beta}{\alpha\beta}\right]$

or $\quad v = v \left[\frac{\alpha+\beta}{\alpha\beta}\right] t \qquad$ to $v = \left(\frac{\alpha\beta}{\alpha+\beta}\right) t$

Total distance covered $= \frac{1}{2}(OB)\ (AM)$

$= \frac{1}{2} \times (t_1 + t_2) \times v$

$= \frac{1}{2} \times t \times v$

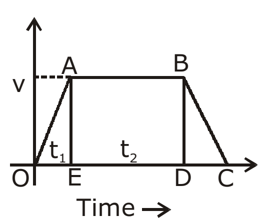
$= \frac{1}{2} \times t \times \left(\frac{\alpha\beta}{\alpha+\beta}\right) t$, using equation

$S = \frac{\alpha\beta}{2(\alpha+\beta)} t^2$

Q.4 The speed of a car increases at a constant rate A from zero to v, and then remains constant for an interval, and finally decreases to zero at a constant rate B. If L is the total distance covered, prove that the total time taken is

$$\frac{L}{v} + \frac{v}{2}\left(\frac{1}{A} + \frac{1}{B}\right)$$

Sol. Graph OA represents the accelerating motion, AB represent the uniform motion (constant speed), and graph BC represents the retarding motion .



$\therefore \quad A = \frac{v}{t_1} \qquad$ or $\quad t_1 = \frac{v}{A}$

$$B = \frac{v}{t_3} \quad \text{or} \quad t_3 = \frac{v}{B}$$

Total distance covered by the car is equal to area under the v-t graph

L = Area OABCO

= Ar OAE + Ar ABDE + Ar BDC

$$= \frac{1}{2}vt_1 + vt_2 + \frac{1}{2}vt_3$$

$$= \frac{1}{2}v(t_1 + t_3) + vt_2$$

or $$\frac{L}{v} = \frac{1}{2}(t_1 + t_3) + t_2$$

$$\frac{L}{v} = \frac{1}{2}\left[\frac{v}{A} + \frac{v}{B}\right] + t_2$$

or $$t_2 = \frac{L}{v} - \frac{v}{2}\left[\frac{1}{A} + \frac{1}{B}\right]$$

$\therefore$ Total time taken $t = t_1 + t_2 + t_3$

$$t = \left(\frac{v}{A} + \frac{v}{B}\right) + \frac{1}{v} - \frac{v}{2}\left[\frac{1}{A} + \frac{1}{B}\right]$$

$$= v\left[\frac{1}{A} + \frac{1}{B}\right] - \frac{v}{2}\left[\frac{1}{A} + \frac{1}{B}\right] + \frac{L}{v}$$

$$= \frac{L}{v} + \frac{v}{2}\left[\frac{1}{A} + \frac{1}{B}\right]$$

Q.5 A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If t is the total elapsed, then calculate :

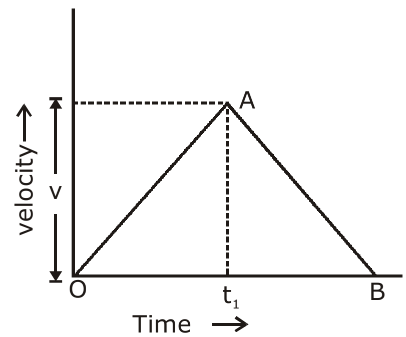
(i) the maximum velocity attained by the car and (ii) the total distance travelled by the car.

Sol. Starting from rest, let the car accelerate for a time t_1. Then, its velocity time graph is a straight line OA sloping upwards. Let v be the maximum velocity attained by the car.

Slope of velocity time graph OA gives the acceleration α.

$$\therefore \quad \alpha = \frac{v}{t_1} \quad \text{or} \quad t_1 = \frac{v}{\alpha} \quad .....(1)$$

After attaining the maximum velocity, the car begins to decelerate. The velocity time graph is AB. Its slope will give the retardation β.



Now, $\beta = \dfrac{v}{t - t_1}$ or $t - t_1 = \dfrac{v}{\alpha}$ .....(2)

Adding (1) and (2), we get $t = v\left(\dfrac{1}{\alpha} + \dfrac{1}{\beta}\right)$

or $v = \dfrac{\alpha\beta}{\alpha + \beta} t$

which gives the maximum velocity attained by the car.

Again, we know that the area under the velocity time graph gives the distance covered.

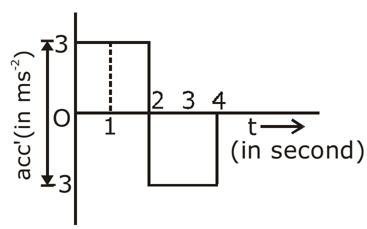
$\therefore$ distance covered = area of ΔOAB

$$= \frac{1}{2} \times t \times v = \frac{1}{2} \times t \times \frac{\alpha\beta}{\alpha+\beta} t = \frac{1}{2} \frac{\alpha\beta}{\alpha+\beta} t^2$$

Q.6 A particle starts from rest at time t = 0 and suffers acceleration as shown in fig. Draw the velocity time graph for the time interval from 0 to 4 second.

Sol. At t = 0, v(0) = 0

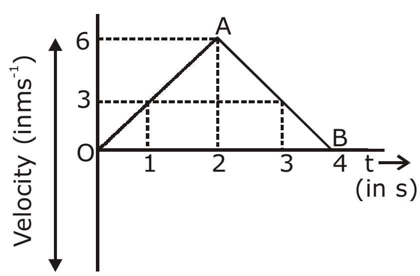
Is is clear from the given graph that acceleration has a constant value of $3ms^{-2}$ from t = 0 to t = 2 second.



During this time interval, the velocity at any instant of time at may be given by

$$v(t) = v(0) + at = 0 + 3t = 3t ms^{-1}$$

This is the equation of a straight line (y = mx) which passes through the origin and has a slope 3. This is represented by the portion OA of the velocity time graph.



At t = 0, v(0) = 0

At t = 1, $v(0) = 3ms^{-1}$

At t = 2, v(0) = 6ms^{-1}

From t = 2s to t = 4s, the acceleration is given to be −3ms^{-2}. Let this time interval be t'.

Again at t' = 0 (t = 2s), v(0) = 6ms^{-1}

Now, the velocity at any instant of time t' is given by

v(t') = v(0) = at' = 6 – 3t' = 6 – 3(t – 2) = 12 – 3t

which is the equation of a straight line whose slope is '−3'.

NUMERICALS :

Q.7 The displacement (s) of a particle moving in one dimensions, under the action of a constant force is related to time t by the equation.

$$t = \sqrt{s} + 3$$

Where s is in meters and t in seconds. Find the displacement of the particle when its velocity is zero.

Sol.

$$t = \sqrt{s} + 3$$

or $$\sqrt{s} = t - 3$$

Squaring both the sides

$$s = t^2 - 6t + 9$$

Differentiating w.r.t. time 't'

$$\frac{ds}{dt} = v = 2t - 6$$

When v = 0 2t – 6 = 0

or t = 3s

$\therefore$ s = (3)^2 – 6(3) + 9

= 9 – 18 + 9 = 0

When velocity of particle is zero, the displacement is also zero.

Q.8 The position(s) of a particle at any time t is given by

$$s = (6t^3 + 4t^2 + t + 7) \text{ m.}$$

Find the instantaneous position, velocity and acceleration of the body at t = 2sec.

Sol. Here $s = 6t^3 + 4t^2 + t + 7$

Differentiating w.r.t time

$$v = \frac{ds}{dt} = 18t^2 + 8t + 1$$

Again differentiating w.r.t. time

$$a = \frac{d^2s}{dt^2} = 36t^3 + 8$$

From equation (1) instantaneous position at t = 2s

$s = 6(2)^3 + 4(2)^2 + 2 + 7$

$= 48 + 16 + 2 + 7 = 73m.$

Instantaneous speed at t = 2s from equation (2)

$v = 18(2)^2 + 8(2) + 1$

$= 89ms^{-1}$

Instantaneous acceleration at t = 2s from equation (3)

$a = 36(2) + 8$

$= 80ms^{-2}$

Q.9 A car moving at $30ms^{-1}$ slows uniformly to a speed of $5ms^{-1}$ in a time of 4s. Calculate :

(i) the acceleration of the car.

(ii) the distance it moves in the third second.

Sol. Here $u = 30ms^{-1}$

$v = 5ms^{-1}$

$t = 4s$

$n = 3$

(i) From Ist equation of motion

$v = u + at$

$$a = \frac{v-u}{t} = \frac{5-30}{4}$$

$= -6.25ms^{-2}$

(-ve sign shows retardation)

(ii) Distance travelled in nth second

$$s_n = u + \frac{a}{2}(2n - 1)$$

$$= 30 + \left[\frac{-6.25}{2}\right](2 \times 3 - 1)$$

$= 14.37m$

Q.10 A car moving along a straight highway with speed of $126kmh^{-1}$ is brought to a stop within a distance of 200m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?

Sol. Initial velocity, $u = 126kmh^{-1}$

$$= 126 \times \frac{5}{18} ms^{-1} = 35ms^{-1}$$

Final velocity, $v = 0$; Distance $S = 200m$

Using $v^2 - u^2 = 2aS$, $0^2 - 35 \times 35 = 2a \times 200$

or $a = -\frac{5}{18} ms^{-2} = -3.06ms^{-2}$

Using $v = u + at$, $0 = 35 - 3.06 \times t$

or $3.06t = 35$

or $t = \frac{35}{3.06} s = 11.4s$

Q.11 The position of an object moving along x-axis is given by $x = a + bt^2$ where $a = 8.5m$, $b = 2.5ms^{-2}$ and t is measured in second. What is its velocity at t = 0s and t = 2s? What is the average velocity between t = 2s and t = 4?

Sol. $v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 2bt = 5.0tms^{-1}$

At $t = 0s$, $v = 0ms^{-1}$ and at $t = 2s$, $v = 10ms^{-1}$

Average velocity = $\frac{x(4) - x(2)}{4 - 2} = \frac{a + 16b - a - 4b}{2} = 6b = 6 \times 2.5 = 15.0ms^{-1}$

Q.12 The engine of an electric train 72m long passes a stationary car with a velocity of $6ms^{-1}$. When the tail end of the train passes the same car, its velocity is $9ms^{-1}$. Calculate the acceleration of the car and the time taken by the train to passes the car.

Sol. $u = 6ms^{-1}$, $v = 9ms^{-1}$, $S = 72m$

Using $v^2 - u^2 = 2aS$, we get $9^2 - 6^2 = 2 \times a \times 72$

or $144a = (15)(3) = 45$ or $a = \frac{45}{144} ms^{-2} = 0.3125ms^{-2}$

Again using $v = u + at$, we get $g = 6 + \frac{45}{144} t$

or $\frac{45}{144} t = 3$ or $t = \frac{45}{144} s = 9.6s$

Q.13 A body travels 200cm in the first two second and 220cm in the next four second. What will be the velocity at the end of seventh second from start?

Sol. At $t = 0$, $x(0) = 0$, $v(0) = u$ (say), $x(t) = 200cm$, $t = 2s$

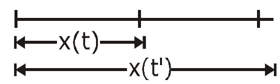
$x(t') = (200 + 220)cm = 420cm$

$t' = (2 + 4)s = 6s$

If a be the uniform acceleration of the particle, then $x(t) = x(0) + v(0)t + \frac{1}{2}at^2$

$$200 = 0 + u + 2 + \frac{1}{2}a \times 4$$

or $100 = u + a$ ....(1)



Again, $x(t') = x(0) + v(0)t' + \frac{1}{2}at'^2$

$$420 = 0 + u \times 6 + \frac{1}{2} \times a \times 36$$

$70 = u + 3a$ ....(2)

Subtracting (1) from (2), we get

$-3a = 2a$ or $a = -15 cms^{-2}$

From equation (1), $u = 100 - (-15) = 115 cms^{-2}$

Now, $t'' = 7s$, $v(t'') = ?$

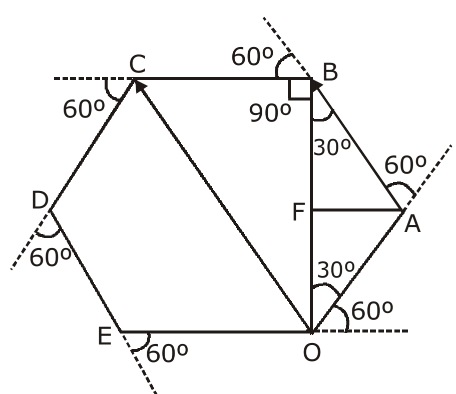
$v(t'') = v(0) + at''$

$\therefore$ $v(t'') = (115 - 15 \times 7)\ cms^{-1} = 10 cms^{-1}$

Q.14 On an open ground, a motorist follows a track that turns to his left by an angle of 60º after every 500m. starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Sol. (i) The path followed by the motorist will be closed hexagonal path.

Suppose the motorist starts his journey from the point O. He takes the third turn at the point C.



Displacement $= \vec{OC}$

Clearly, $OC = \sqrt{OB^2 + BC^2} = \sqrt{(OF + FB)^2 + BC^2}$

$$= \sqrt{(500\cos 30º + 500\cos 30º)^2 + 500^2}$$

$$= \sqrt{\left(2 \times 500 \times \frac{\sqrt{3}}{2}\right)^2 + 500^2}$$

$$= 500\sqrt{4}\text{m} = 1000\text{m} = 1\text{m}$$

Total path length = 500m + 500m + 500m

= 1500m = 1.5km

$$\frac{\text{Magnitude of displacement}}{\text{Total path length}} = \frac{1\text{km}}{1.5\text{km}} = \frac{2}{3} = 0.67$$

(ii) The motorist will take the sixth turn at O. Displacement is zero.

Path length is 3000m, i.e., 3km

Ratio of magnitude of displacement and path length is zero.

(iii) The motorist will take the 8th turn at B.

Magnitude of displacement = 2 × 500 cos30º = $500\sqrt{3}\,\text{m} = \frac{\sqrt{3}}{2}\text{km}$

Path length = 8 × 500m = 4km

Ratio of magnitude of displacement and path length is $\frac{\sqrt{3}/2}{4}$, i.e. $\frac{\sqrt{3}}{8}$, i.e. 0.22