Problems Based on Acceleration

VERY SHORT ANSWER TYPE QUESTIONS :

Q.1 Can a body have an acceleration with zero velocity.

Sol. Yes. When a body is thrown vertically up then at the highest point of its motion the velocity is zero but acceleration is not zero.

Q.2 What can be determined from -

(a) Displacement time graph (b) velocity time graph

Sol. (a) The slope of displacement time plot provides us the velocity of object.

(b) The area below velocity time plot provides the displacement and the slope of it gives the acceleration.

Q.3 Write the S.I. unit and the dimension of acceleration.

Sol. m/sec^2, $M^0\ L^1\ T^{-2}$.

Q.4 Which physical quantity can be calculated from the slope of velocity time graph.

Sol. Acceleration.

Q.5 What is SLAP.

Sol. The time rate of acceleration is called SLAP. It's unit is m/s^{-3}.

Q.6 What do you mean by retardation?

Sol. When the velocity of a body is decreasing steadily, it has negative acceleration. Negative acceleration is also called retardation.

Q.7 Is the kinematic equation $S = ut + \frac{1}{2}at^2$ true if acceleration is not constant?

Sol. No. All kinematic equations are true only if acceleration is constant.

Q.8 Give position-time relation for a uniformly accelerated motion.

Sol. $s(t_2) - s(t_1) = v\ (t_1)\ (t_2 - t_1) + \frac{1}{2}\ a\ (t_2 - t_1)^2$.

or $s(t) - s(0) = v\ (0)\ t + \frac{1}{2}\ at^2$.

Q.9 In case of uniform linear motion, what is the value of acceleration.

Sol. zero.

Q.10 Why is time stated twice in stating acceleration?

Sol. Acceleration is the double time rate of change of displacement.

Problems Based on Acceleration

SHORT ANSWER TYPE - I QUESTIONS :

Q.11 Define acceleration.

Sol. The rate of change of velocity of a particle with respect to time is called acceleration. It is measured by the ratio of change in velocity to the corresponding change in time i.e

$$a = \frac{v(t_2) - v(t_1)}{t_2 - t_1}$$

Q.12 Define instantaneous acceleration.

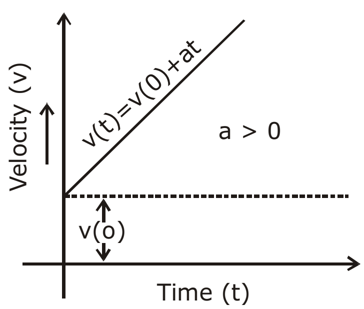
Sol. The instantaneous acceleration of a moving body is defined as the acceleration of the body at a particular instant of time.

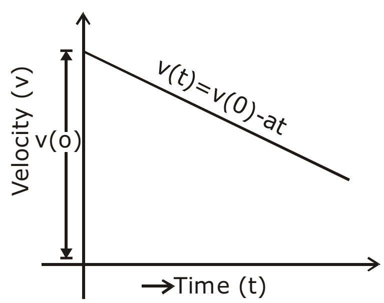
$$\text{Instantaneous acceleration } a = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}$$

$$= \frac{dV}{dt}$$

Q.13 Plot a velocity time graph for a body having uniformly accelerated motion.

Sol.





Q.14 Write the equations of motion with uniform acceleration.

Sol. (i) $v(t) = v(0) + at$

(ii) $s(t) - s(0) = v(0)t + \frac{1}{2} at^2$

(iii) $v^2(t) = v^2(0) + 2a[s(t) - s(0)]$

Q.15 Derive an expression for stopping distance of a vehicle in terms of initial velocity v_0 and deceleration a.

Sol. Let d_s be the distance travelled by a vehicle before it stops.

Using $v^2 - u^2 = 2aS$,

we get $0^2 - v_0^2 = -2ad_s$ or $d_s = \frac{v_0^2}{2a}$

The stopping distance is proportional to the square of the initial velocity. Doubling the initial velocity increases the stopping distance by a factor of 4, provided deceleration is kept the same.

Problems Based on Acceleration

Q.16 Differentiate between uniform and variable acceleration.

Sol. When the velocity of a body changes by equal amounts in equal intervals of time, then the acceleration is called uniform acceleration.

In case of non-uniform acceleration, the velocity of body changes by unequal amounts in equal intervals of time.

SHORT ANSWER TYPE - II QUESTIONS :

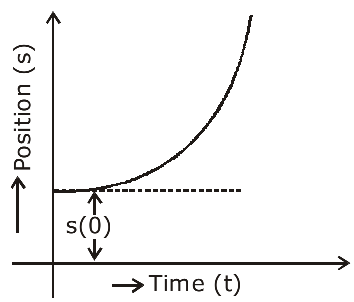
Q.17 Plot position time graph for a body having uniformly accelerated motion if

(i) $a > 0$ (ii) $a < 0$

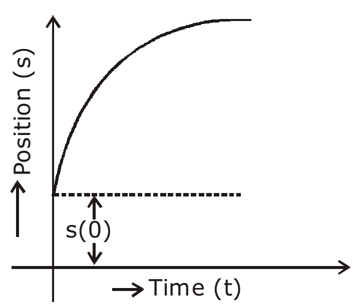
Sol. In case of uniformly accelerated motion, the displacement is given by

$$s(t) - s(0) = v(0)\,t + \frac{1}{2}at^2$$

Case (i) when $a > 0$



Case (ii) when $a < 0$ $\quad s(t) - s(0) = v(0)t - \frac{1}{2}at^2$



Q.18 The distance covered by the body is found to be directly proportional to the square of time. Is the body moving with uniform velocity or with uniform acceleration.

Sol. Given $s \propto t^2$

or $s = kt^2$

Where k is proportionality constant.

Differentiating w.r.t time

$$\frac{ds}{dt} = v = 2kt$$

Again differentiating w.r.t. time

$$\frac{d^2s}{dt^2} = a = 2k$$

or $\quad a = \text{constant}$

Thus the body is moving with constant acceleration.

Q.19 Derive an expression for the distance travelled by a uniformly accelerated body in n^{th} second.

Sol. Let us consider a particle moving with constant accleration along the positive direction of x-axis. S_n and S_{n-1} be the distance covered by the body in n and n–1 seconds respectively.

$$\therefore \quad S_n = un + \frac{1}{2}an^2$$

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

The distance covered during nth second of motion.

$$S_{nth} = S_n - S_{n-1}$$

$$= (un + \frac{1}{2}an^2) - [u(n-1) + \frac{1}{2}a(n-1)^2]$$

$$= un + \frac{1}{2}an^2 - u(n-1) + \frac{1}{2}a(n-1)^2$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}a(n^2 - 2n + 1)$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + an - \frac{1}{2}a$$

$$= u - \frac{a}{2} + an$$

$$= u + \frac{a}{2}(2n - 1)$$

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

Problems Based on Acceleration

Q.20 Establish the relation

$$v(t_2) = v(t_1) + a(t_2 - t_1)$$

where the letters have their usual meaning.

Sol. Let us consider a particle is moving with constant acceleration along the positive direction of x-axis. $v(t_1)$ and $v(t_2)$ be the velocities of the particle at times t_1 and t_2 respectively.

$\therefore$ Acceleration $a = \dfrac{\text{Change in velocity}}{\text{Change in time}}$

$$a = \frac{v(t_2) - v(t_1)}{(t_2 - t_1)}$$

or $v(t_2) - v(t_1) = a(t_2 - t_1)$

or $v(t_2) = v(t_1) + a(t_2 - t_1)$

Q.21 Derive second equation of motion i.e.

$$s(t_2) - s(t_1) = v(t_1)(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2.$$

Sol. Consider a particle moving with uniform acceleration. $v(t_1)$ and $v(t_2)$ represent the velocities of the particle at time $t = t_1$ and $t = t_2$ respectively. The magnitude of average velocity during this time interval is given by

$$v_{av} = \frac{v(t_1) + v(t_2)}{2}$$

$$= \frac{1}{2}[v(t_1) + v(t_1) + a(t_2 - t_1)] \text{ using 1st equation of motion}$$

$$= v(t_1) + \frac{a}{2}(t_2 - t_1)$$

Let $s(t_1)$ and $s(t_2)$ are the position coordinates of the particle at time t_1 and t_2 respectively.

$\therefore$ $s(t_2) - s(t_1) = v_{av} \times$ time interval.

$$s(t_2) - s(t_1) = \left[v(t_1) + \frac{a}{2}(t_2 - t_1)\right] \times (t_2 - t_1)$$

or $$s(t_2) - s(t_1) = \left[v(t_1) + \frac{a}{2}(t_2 - t_1)\right] \times (t_2 - t_1)$$

Q.22 Obtain a relation between initial velocity $v(t_1)$ and final velocity $v(t_2)$ of uniformly accelerated body after it has made a certain displacement.

Sol. Let $s(t_1)$ and $s(t_2)$ are the position coordinates of a particle at time t_1 and t_2 respectively $v(t_1)$ and $v(t_2)$ be the velocities of particle at these times.

$\therefore$ The magnitude of average of velocity of the particle from time t_1 to t_2

$$v_{av} = \frac{v(t_1) + v(t_2)}{2}$$

Also $s(t_2) - s(t_1) = v_{av} \times (t_2 - t_1)$

$$s(t_2) - s(t_1) = \left[\frac{v(t_1) + v(t_2)}{2}\right]\left[\frac{v(t_2) - v(t_1)}{a}\right]$$ (using 1st equation of motion)

$\therefore$ $$s(t_2) - s(t_1) = \frac{v^2(t_2) - v^2(t_1)}{2a}$$

or $v^2(t_2) - v^2(t_1) = 2a\,[s(t_2) - s(t_1)]$

NUMERICALS :

Q.23 An automobile moving at a speed $20ms^{-1}$ is accelerated at the rate of $2ms^{-2}$. Calculate the velocity after 5sec.

Sol. $u = 20ms^{-1}$

$a = 2ms^{-2}$

$t = 5sec.$

From 1st equation of motion

$V = u + at = 20 + 2 \times 5 = 30ms^{-1}$

Q.24 A car travelling at a speed of $15ms^{-1}$ is accelerated at the rate of $2ms^{-1}$. Find out the velocity after 5 seconds.

Sol. Here $u = 15ms^{-1}$

$a = 2ms^{-2}$ $t = 5sec.$

$v = ?$

from formula $v = u + at$

$= 15 + 2 \times 5 = 25ms^{-1}$

Q.25 A car moving along a straight highway with speed of $126kmh^{-1}$ is brought to a stop with in a distance of 200m. What is the retardation of the car and how long does it take for the car to stop.

Sol. Here $u = 126kmh^{-1}$

$$= 125 \times \frac{5}{8} = 135ms^{-1}$$

From third equation of motion

$v^2 = u^2 + 2as$

$o^2 = (35)^2 + 2a \times 200$

$$\therefore \quad a = -\frac{35 \times 35}{2 \times 200} = 3.06ms^{-2}$$

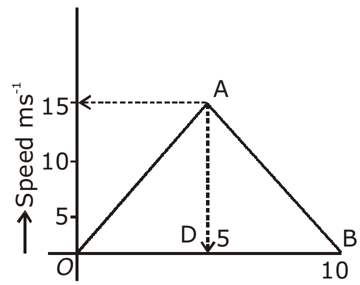
Now $v = u + at$

$0 = 35 - (3.06)t$

$$\therefore \quad t = \frac{35}{3.06} = 11.4sec$$

Problems Based on Acceleration

Q.26 The speed time graph of particle moving along a fixed direction is shown. Calculate the acceleration of the particle between 5s to 10s.



Sol. Acceleration of the particle equals to slope of velocity time graph

$\therefore$ acceleration a = slope of graph AB

$$= \frac{AB}{DB}$$

$$= \frac{0-15}{10-15}$$

$$= -3ms^{-1}$$

Q.27 The reaction time for an automobile driver is 0.7 second. If the automobile can be decelerated at $5ms^{-2}$, calculate the total distance travelled in coming to stop from an initial velocity of $30kmh^{-1}$, after a signal is observed.

Sol. Since the reaction time of the driver is 0.7 second therefore the automobile, during this time, will continue to move with uniform velocity of $30kmh^{-1}$,

i.e., $30 \times \frac{5}{18} ms^{-1}$ or $\frac{25}{3} ms^{-1}$

Distance covered during 0.7 second = $\frac{25}{3} \times 0.7m = 5.83m$

Let us choose that time as reference time (t = 0) when the automobile begins to decelerate.

So, at t = 0, x(0) = 0, v(0) = $\frac{25}{3} ms^{-1}$, a = $-5\ ms^{-2}$, x(t) = ?, v(t) = 0

$0^2 - \left(\frac{25}{3}\right)^2 = 2(-5)\ [x(t) - 0]$ or $x(t) = \frac{625}{9} \times \frac{1}{10} m = 6.94m$

Total distance travelled = 5.83m + 6.94m = 12.77m

Problems Based on Acceleration

Q.28 A three wheeler starts from rest, accelerates uniformly with $1ms^{-2}$ on a straight road for 10s, then moves with uniform velocity. Plot the distance covered by the vehicle during the nth second (n = 1, 2, 3....) versus n. What do you expect this plot to during accelerated motion: a straight line or a parabola?

Sol. Here $u = 0$

$a = ms^{-2}$

Distance covered in nth second is given by

$$s_n = u + \frac{a}{2}(2n - 1)$$

$$= 0 + \frac{1}{2}(2n - 1)$$

From graph (i) straight line AB shows uniformly accelerated motion.

(ii) After 10seconds the motion is uniform, shown by the straight line BC parallel to the time axis.

n in sec.	S_n in meter
1	0.5
2	1.5
3	2.5
4	3.5
5	4.5
6	5.5
7	6.5
8	7.5
9	8.5
10	9.5