PROJECTILE MOTION

DEFINITION

An object is thrown with an initial speed (say u) in any direction (say θ from horizontal) and then a constant force F = mg continuously acts on it in vertically downward direction. Due to this, body moves freely under gravity. This motion is called projectile motion.

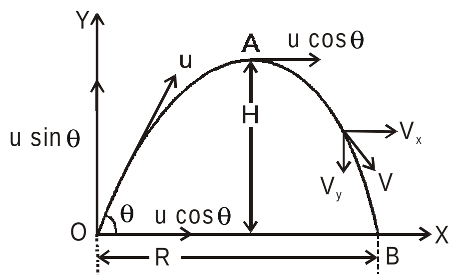
ASSUMPTION

(i) It is assumed that acceleration due to gravity is constant

(ii) The effect of air resistance is negligible.

(iii) Range is small, so that earth can be considered flat.

PROJECTILE THROWN AT AN ANGLE WITH HORIZONTAL



INITIALLY GIVEN PARAMETERS

POINT OF PROJECTION : The point O from where projectile is thrown.

ANGLE OF PROJECTION (θ) : The angle from horizontal, towards which projectile is thrown.

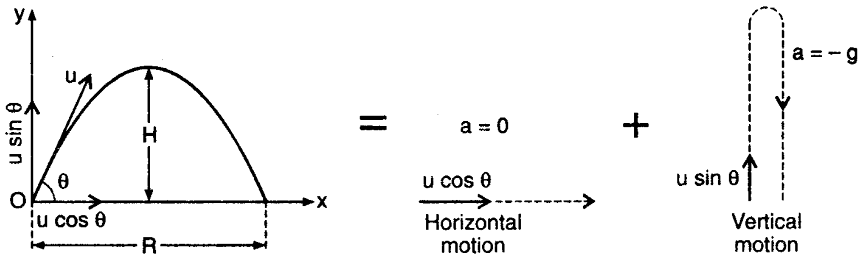
INITIAL SPEED OF PROJECTION (u) : The speed with which projectile is thrown.

The projectile motion near the surface of earth consists of superposition of two simultaneous independent motions:

1. Horizontal motion at *constant* horizontal *speed* u cos θ and horizontal acceleration = 0 as no horizontal force is present.
2. Vertical motion with varying vertical speed and *constant acceleration* due to gravity=-g due to downward gravity force.
3. Thus, here also we can apply law of independence of direction and solve the problems by dividing one 2-D motion into two 1-D motions.

LAW OF INDEPENDENCE OF DIRECTIONS

Components of $\vec{v}$, $\Delta\vec{r}$, $\vec{a}$ which are perpendicular to each other are always independent of each other, so x, v_x, a_x are independent of y, v_y, a_y.



parabolic motion = horizontal motion + vertical motion

(1) GENERAL EQUATIONS FOR HORIZONTAL MOTION

horizontal component of initial speed = $u_x = u \cos\theta$

horizontal acceleration = $a_x = 0$

horizontal velocity at any time t = $v_x = u_x + 0(t) = u_x = u \cos\theta$

i.e., horizontal component of initial velocity is constant at all instants.

instantaneous horizontal position of the projectile

$$\Rightarrow \quad x = u_x t + \frac{1}{2} a_x t^2 = u \cos\theta \; t + \frac{1}{2}(0)t^2 = u \cos\theta \; t$$

Ex. *If air resistance is not considered in projectiles, the horizontal motion takes place with :*

(A) Constant velocity *(B) Constant acceleration*

(C) Constant retardation *(D) Variable velocity*

Sol. (A)

In the absence of air resistance, the projectile moves with constant horizontal velocity because acceleration due to gravity is totally vertical.

(2) GENERAL EQUATIONS FOR VERTICAL MOTION

vertical component of initial speed = $u_Y = u \sin\theta$

vertical acceleration = $a_Y = -g$ (as force of gravitation is always downward)

vertical velocity at any time t = $v_Y = u_Y + a_Y t = u \sin\theta - gt$

vertical velocity at any height y $\Rightarrow v_y^2 = u_y^2 + 2a_y y = u^2 \sin^2\theta - 2gy$

Vertical component of velocity is positive, when particle is moving up.

Vertical component of velocity is zero, when particle is at the highest point of trajectory. Vertical component of velocity is negative, when particle is coming down.

Instantaneous vertical position of the projectile $\Rightarrow y = u_y t + \frac{1}{2} a_y t^2 = u \sin\theta \; t - \frac{1}{2} g t^2$

Ex.1 *When a projectile is fired at an angle θ w.r.t. horizontal with velocity u, then its vertical component:*

(A) Remains same *(B) Goes on increasing with height*

(C) Goes on decreasing with height *(D) First increases then decreases with height*

Sol. (C)

The vertical component goes on decreasing and eventually becomes zero.

Ex.2 *In the previous example, the horizontal component ignoring air resistance :*

(A) Remains same *(B) Goes on increasing with height*

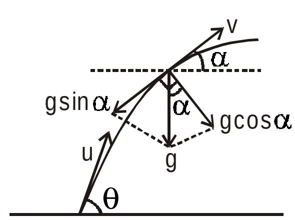
(C) Goes on decreasing with height *(D) First increases then decreases with height*

Sol. (A)

Because there is no acceleration or retardation along horizontal direction, hence horizontal component of velocity remains same.

	GENERAL EQUATIONS	HORIZONTAL MOTION	VERTICAL MOTION
(1)	component of ini. speed	$u_x = u\cos\theta$	$u_y = u\sin\theta$
(2)	acceleration	$a_x = 0$	$a_y = -g$
(3)	velocity at any instant	$v_x = u_x = u\cos\theta$	at any time t, $v_y = u\sin\theta - gt$ at any height y $v_y^2 = u^2\sin^2\theta - 2gy$
(4)	position at any instant	$x = u\cos\theta\ t$	$y = u\sin\theta\ t - \frac{1}{2}gt^2$

(3) POSITION OF PARTICLE AT ANY MOMENT = $(x,y) = (u\cos\theta\ t\ ,\ u\sin\theta\ t - \frac{1}{2}gt^2)$



(4) NORMAL AND TANGENTIAL ACCELERATION AT ANY MOMENT

'g' can be split into two parts, "g cos α" normally inward to the curve and "g sin α" tangentially opp. to the direction of motion, where α is the angle tangent makes with horizontal at any instant (note that α is not the angle of projection θ which is a constant entity, α decreases with ascent of projectile).

(5) RESULTANT VELOCITY OF PARTICLE V AT ANY MOMENT

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2} = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

$\therefore$ Resultant velocity goes on decreasing during the first half of motion and becomes minimum at max. height.

$v_{min} = u_x = u\cos\theta$ at max. height.

at an angle α from horizontal i.e. $\tan\alpha = \frac{v_y}{v_x} = (u\sin\theta - gt)/u\cos\theta$

Ex. *The point from where a ball is projected is taken as the origin of the coordinate axes. The x and y components of its displacement are given by x = 6t and y = 8t – 5t². What is the velocity of projection?*

(A) 6 ms⁻¹ *(B) 8 ms⁻¹* *(C) 10 ms⁻¹* *(D) 14 ms⁻¹*

Sol. (C)

$$v_x = \frac{dx}{dt} = 6 \text{ and } v_y = \frac{dy}{dt} = 8 - 10t = 8 - 10 \times 0 = 8$$

$$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10 \text{ ms}^{-1}.$$

(6) |DISPLACEMENT| FROM ORIGIN (S) : $S = \sqrt{x^2 + y^2} = \sqrt{(u\cos\theta\, t)^2 + (u\sin\theta\, t - \frac{1}{2}gt^2)^2}$

(7) TIME OF FLIGHT (T) :

The total time taken by the projectile to go up and come down to the same level from where it was projected is called time of flight.

Time taken to reach the highest point (where vertical component of velocity $v_Y = 0$)

$0 = u \sin\theta - gt$, i.e. $t = (u \sin\theta / g)$ (using $v_y = u_y + a_y t$)

Now as time taken to go up is equal to the time taken to come down, so

$$T = 2t = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$$

$\therefore$ Time of flight depends on vertical component of initial velocity only.

(8) HORIZONTAL RANGE (R) :

(a) It is the horizontal distance travelled by a body during the time of flight.

So R = displacement x in time T = $s_x = u \cos\theta\ T + 1/2\ (0)\ t^2$

(using $s_x = u_x t + \frac{1}{2} a_x t^2$)

we get, $R = u \cos\theta \times (2u \sin\theta / g)$

i.e. $R = \frac{u^2 \sin 2\theta}{g}$ [as $2 \sin\theta \cos\theta = \sin 2\theta$]

or $R = \frac{2u_x u_y}{g}$

$\therefore$ Range depends on both components of initial velocity.

(b) If angle of projection is changed from θ to $\theta' = (90 - \theta)$, the range remains unchanged

$$R' = \frac{u^2 \sin 2\theta'}{g} = \frac{u^2 \sin[2(90-\theta)]}{g} = \frac{u^2 \sin 2\theta}{g} = R$$

(c) For range to be maximum, (for a given value of u, g)

$R \propto \sin(2\theta)$

as $(\sin 2\theta)_{max} = 1$ at $\theta = 45^o$

$\therefore$ R_{max} is at $\theta = 45^o$

$R_{max} = (u^2/g)$

When the range is maximum, the height H reached by the projectile

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

i.e., if a person can throw a projectile to a maximum distance R_{max}, the maximum height to which it will rise is $(R_{max}/4)$ and the angle of projection is 45^o

Ex.1 *A bullet is fired from a cannon with velocity 500 m/s. If the angle of projection is 15º and g = 10 m/s², then the range is :*

(A) 25×10^3 m *(B) 12.5×10^3 m* *(C) 50×10^2 m* *(D) 25×10^2 m*

Sol. (B)

Ex.2 *A ball is thrown upwards at an angle of 60º to the horizontal. It falls on the ground at a distance of 90 m. If the ball is thrown with the same initial velocity at an angle 30º, it will fall on the ground at a distance of :*

(A) 120 m *(B) 90 m* *(C) 60 m* *(D) 30 m*

Sol. (B)

Ex.3 *A stone is thrown with a velocity u making an angle θ with the horizontal. The horizontal distance covered by its fall to ground is maximum when the angle θ is equal to :*

(A) 0º *(B) 30º* *(C) 45º* *(D) 90º*

Sol. (C)

(9) MAXIMUM HEIGHT (H) :

(a) It is the maximum height from the point of projection a projectile can reach.

At this point, vertical component of velocity is equal to 0

Using $v_y^2 = u_y^2 + 2as$

for vertical motion i.e., with v= 0 and u = u $\sin\theta$ and a =-g, y = H, it becomes

$$0 = (u \sin\theta)^2 - 2gH$$

i.e. $H = \frac{u^2 \sin^2 \theta}{2g}$

or $H = \frac{u_y^2}{2g}$

$\therefore$ max. height depends on vertical component of initial velocity only.

Ex.1 *For a projectile fired at an angle θ with the horizontal with velocity u, the maximum height attained by it is given by :*

(A) $\frac{u^2 \sin\theta}{2g}$ (B) $\frac{u^2 \sin^2\theta}{2g}$ (C) $\frac{u^2 \sin^2\theta}{g}$ (D) $\frac{u^2 \cos^2\theta}{2g}$

Sol. (B)

Ex.2 *If a projectile is fired at an angle θ with the vertical with velocity u, then maximum height attained is given by :*

(A) $\frac{u^2 \sin\theta}{2g}$ (B) $\frac{u^2 \sin^2\theta}{2g}$ (C) $\frac{u^2 \sin^2\theta}{g}$ (D) $\frac{u^2 \cos^2\theta}{2g}$

Sol. (D)

(b) At maximum height, velocity is horizontal and acceleration is vertically downward, so

$\vec{a}.\vec{v} = 0$ at max. height.

(c) θ to get maximum possible value of H for given u :

$H = \frac{u^2 \sin^2 \theta}{2g}$, $\sin^2\theta$ = max = 1, i.e., $\theta = 90^\circ$

$\therefore$ $H \propto \sin^2\theta$. So as max($\sin^2\theta$) = 1 at $\theta = 90^\circ$

Hence, $H_{max} = \frac{u^2 \sin^2 90}{2g} = \frac{u^2}{2g}$

i.e., a projectile will have height H maximum, when it is projected vertically (i.e., $\theta = 0^\circ$) and maximum height reached will be ($u^2/2g$). When the height H is maximum, the range will be

$$R = \frac{u^2 \sin 180}{g} = 0$$

Further, from Eqns. (10) and (12),

$$H_{max} = (R_{max}/2)$$

i.e., if a person can throw a projectile to a maximum distance R_{max} [$= u^2/g$ with $\theta = 45^o$], the maximum height to which, he can throw the projectile is $H_{max} = (R_{max}/2)$ [$=(u^2/2g)$ with $\theta = 90^o$].

Ex.1 *During a projectile motion if the maximum height equals the horizontal range, then the angle of projection with the horizontal is :*

(A) $\tan^{-1}(1)$ *(B) $\tan^{-1}(2)$* *(C) $\tan^{-1}(3)$* *(D) $\tan^{-1}(4)$*

Sol. (D)

$$H = \frac{u^2 \sin^2\theta}{2g} \text{ and } R = \frac{u^2 \sin 2\theta}{g}$$

Since $H = R$

$$\frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \times 2\sin\theta\cos\theta}{g}$$

or $\tan\theta = 4$ or $\theta = \tan^{-1}(4)$

Ex.2 *If R is the maximum horizontal range of a particle, then the greatest height attained by it is :*

(A) R *(B) 2R* *(C) $\frac{R}{2}$* *(D) $\frac{R}{4}$*

Sol. (D)

$$R = \frac{u^2}{g} \text{ and } H = \frac{u^2 \sin^2\theta}{2g}$$

For the maximum range, $\theta = 45^o$

$$\therefore \quad H = \frac{u^2 \sin^2 45^o}{2g} = \frac{u^2}{4g} = \frac{R}{4}$$

Ex.3 *A particle is projected with a velocity v, so that its range on a horizontal plane is twice the greatest height attained. If g is acceleration due to gravity, then its range is :*

(A) $\frac{4v^2}{5g}$ (B) $\frac{4g}{5v^2}$ (C) $\frac{4v^3}{5g^2}$ (D) $\frac{4v}{5g^2}$

Sol. (A)

$$H = \frac{v^2 \sin^2\theta}{2g} \text{ and } R = \frac{v^2 \sin 2\theta}{g}$$

Since R = 2H, so $\frac{v^2 \sin 2\theta}{g} = 2 \times \frac{v^2 \sin^2\theta}{2g}$

or 2 sin θ cos θ = $\sin^2\theta$ or tan θ = 2

∴ $R = v^2 \times \frac{2}{g} \times \sin\theta \cos\theta$

$= \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$

Ex.4 *If a projectile just crosses a wall of height h then what is the time of flight.*

Sol. $h = u_y T/2 - 1/2g\ T^2/4$ (for half motion)

$0 = u_y T - 1/2gT^2$ (for full motion)

On solving both equations : $T = 2\sqrt{\frac{2h}{g}}$

(10) TRAJECTORY

It is the path followed by the projectile i.e. a curve joining positions of the same particle in motion at different instants of time.

(The curve joining position of different particles in motion at same instant is called profile.)

$x = u\cos\theta\ t$ ∴ $t = \frac{x}{u\cos\theta}$

$y = u\sin\theta\, t - \frac{1}{2}gt^2$

$y = u\sin\theta \left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$

$y = (\tan\theta)\,x - \frac{g}{2u^2\cos^2\theta}x^2$

This is an equation of the type, $y = ax - bx^2$, which represents a parabola; so the path of a projectile is a parabola.

Also, $y = x\tan\theta\left[1 - \frac{x}{R}\right]$

$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$

NOTE : This eq^n is quadratic in both tan θ and x.

Equation of trajectory as quadratic equation of tan α is

$$\tan^2 \alpha - \frac{2u^2}{gx} \tan \alpha - \left(\frac{2u^2 y}{gx^2} - 1\right) = 0$$

Real roots mean point (x, y) lie in the projectile path. Complex roots imply that point (x, y) is out of range.

Ex. *The equation of the trajectory of an oblique projectile is*

$$y = \sqrt{3}\, x - \frac{1}{2} g x^2$$

Here, x and y are in metre and g is in m/s^2. The angle of projection is :

(A) 0º *(B) 90º* *(C) 45º* *(D) $\tan^{-1}\sqrt{3}$*

Sol. (D)

(11) CHANGE IN MOMENTUM (during complete flight)

Initial velocity $\vec{u}_i = u \cos\theta\, \hat{i} + u \sin\theta\, \hat{j}$

Final velocity $\vec{u}_f = u \cos\theta\, \hat{i} - u \sin\theta\, \hat{j}$

Change in velocity for complete motion, $\Delta\vec{u} = \vec{u}_f - \vec{u}_i = -2u \sin\theta\, \hat{j}$

Change in momentum for complete motion, $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{u}_f - \vec{u}_i)$

$\Delta\vec{p} = m(-2u \sin\theta)\hat{j} = -2mu \sin\theta\, \hat{j} = mgT(-\hat{j})$

or $\Delta\vec{p} = -mgT\hat{j}$

(12) KINETIC ENERGY AND POTENTIAL ENERGY AT ANY INSTANT T

KE = 1/2 mv^2 = 1/2 m $[u^2 \cos^2\theta + (u \sin\theta - gt)^2]$

Velocity is minimum at highest point, which equals to horizontal component u cosθ, so

KE (min) = 1/2 m $(u \cos\theta)^2$ = 1/2 m $u^2 \cos^2\theta$ = $KE_O \cos^2\theta$

Where, KE_O is initial kinetic energy = 1/2mu^2

Potential energy at any instant = mg y = mg (u sinθ t - $\frac{1}{2}gt^2$)

$$= mg\, x \tan\theta \left[1 - \frac{x}{R}\right]$$

Potential energy will be max at highest point and equal to

$$(PE)_H = mgH = mg\,\frac{u^2 \sin^2\theta}{2g} = \frac{1}{2} mu^2 \sin^2\theta$$

so, $(PE)_H + (KE)_H = \frac{1}{2} mu^2 (\sin^2\theta + \cos^2\theta) = \frac{1}{2} mu^2$

Which is the ME at the point of projection. So, in projectile motion mechanical energy is conserved. Furthermore,

$$\left[\frac{PE}{KE}\right]_H = \frac{(1/2)mu^2 \sin^2\theta}{(1/2)mu^2 \cos^2\theta} = \tan^2\theta$$

So if $\theta = 45^o$, $PE = KE = \left(\frac{1}{2}\right) ME$ (at top most point).

i.e., if a body is projected at an angle of 45^o to the horizontal then at the highest point, half of its mechanical energy is kinetic and half potential.

Ex.1 *A ball is thrown upwards and it returns to ground describing a parabolic path. Which of the following quantities remains constant throughout the motion ?*

(A) Kinetic energy of the ball *(B) Speed of the ball*

(C) Horizontal component of velocity *(D) Vertical component of velocity.*

Sol. (C)

Horizontal component of velocity remains constant throughout the motion, as it is not affected by acceleration due to gravity which is directed vertically downwards.

Ex.2 *A ball whose kinetic energy is E, is thrown at an angle of 45º with the horizontal. Its kinetic energy at the highest point of its flight will be :*

(A) E *(B)* $\frac{E}{\sqrt{2}}$ *(C)* $\frac{E}{2}$ *(D) Zero*

Sol. (C)

At the highest point, $v = u \cos 45^o = \frac{u}{\sqrt{2}}$

Hence, K (at the top) $= \frac{1}{2} m \left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}\left[\frac{1}{2} mu^2\right] = \frac{E}{2}$

Ex.3 *A particle is fired with speed u making angle θ with the horizontal. Its potential energy at the highest point is :*

(A) $\frac{1}{2}mu^2 \sin^2\theta$ (B) $\frac{1}{2}mu^2 \cos^2\theta$ (C) $\frac{1}{2}mu^2$ (D) $\frac{1}{2}mu^2 \sin^2 2\theta$

Sol. (A)

The vertical component of velocity is reduced to zero. The corresponding kinetic energy is converted into potential energy (law of conservation of energy).

$$\therefore \quad PE = \frac{1}{2}m(u\sin\theta)^2 = \frac{1}{2}mu^2\sin^2\theta.$$

(13) RADIUS OF CURVATURE AT ANY POINT ON THE PATH OF A PROJECTILE

Consider a particle moving along any curve (may be parabola or circle or any other). At any instant t, let its velocity vector v is making an angle α with the horizontal. We choose tangential axis and normal axis as shown in fig.

Centripetal acceleration of particle is directed towards normal axis. Component of g towards normal axis provides centripetal acceleration.

As $a_c = \frac{v^2}{R_c}$ (where a_c = centripetal acceleration , R_c = radius of curvature).

$$\therefore R_c = \frac{(\text{instantaneous velocity})^2}{\text{centripetal acceleration}}$$

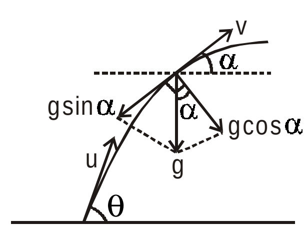
(i) Radius of curvature in terms of t :

Radius of curvature $R_c = \frac{v^2}{a_c} = \frac{v^2}{g\cos\alpha}$

As

$$v^2 = v_x^2 + v_y^2 = (u\cos\theta)^2 + (u\sin\theta - gt)^2 = u^2 + g^2t^2 - 2ugt\sin\theta$$

and $\tan\alpha = \frac{v_y}{v_x} = \frac{(u\sin\theta - gt)}{u\cos\theta}$



$$\therefore R_c = \frac{u^2 + g^2t^2 - 2ugt\sin\theta}{g\cos\alpha}$$

where, $\alpha = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$

(ii) Radius of curvature in terms of y

v and α can be calculated in terms of θ and y

$v_y^2 = (u \sin \theta)^2 - 2gy$

$v_x = u \cos \theta$

$\therefore$ $v^2 = u^2\cos^2\theta + u^2\sin^2\theta - 2gy = u^2 - 2gy$

$$\tan \alpha = \frac{v_y}{v_x} = \frac{\sqrt{(u\sin\theta)^2 - 2gy}}{u\cos\theta}$$

$$\therefore \quad R_c = \frac{u^2 - 2gy}{g\cos\alpha}$$

where $\alpha = \tan^{-1}\left(\frac{\sqrt{(u\sin\theta)^2 - 2gy}}{u\cos\theta}\right)$

Equation of Radius of curvature in general = $R_c = \dfrac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$

Ex. *A projectile is thrown at an angle of 45º with the ground. At the highest position, the radius of curvature of the projectile is-*

Sol. $R_c = \dfrac{v^2}{a_c} = \dfrac{(u\cos 45^\circ)^2}{g\cos 0^\circ} = \dfrac{u^2}{2g}$

SPECIAL FIND OUTS OF PROJECTILE MOTION

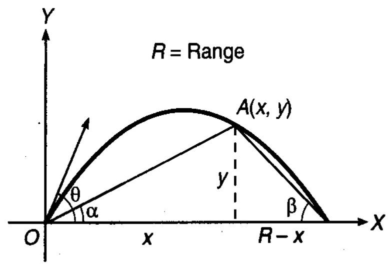
(1) Range is same for two angles, θ and $(90 - \theta)$.

(2) Sum of heights at angles θ and $(90 - \theta)$ is independent of θ, it is $H_\theta + H_{90-\theta} = \dfrac{U^2}{2g}$ and $\dfrac{H_\theta}{H_{90^\circ-\theta}} = \tan^2\theta$.

(3) $H = \dfrac{1}{4} R \tan\theta$ or $R = 4H \cot\theta$. If range is n times of H, then $\tan\theta = \dfrac{4}{n}$.

(for θ = 45º, R is 4 times H)

(4) $\tan\theta = \tan\alpha + \tan\beta$. If B is the highest point, then $\alpha = \beta$ and then $\tan \alpha_H = \frac{1}{2} \tan \theta$.



(5) The greatest height to which man can throw a stone is H. The greatest distance upto which he can throw the stone is 2H.

$\therefore$ $H = \frac{1}{8} gT^2$ (H = max. height, T = time of flight)

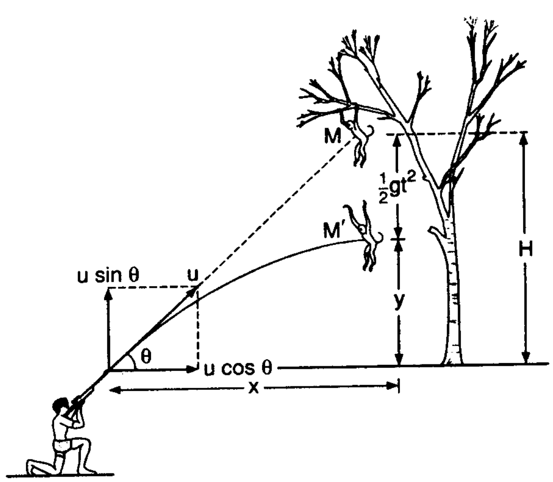
CONCEPTS REGARDING PROJECTILE MOTION (SAME HORIZONTAL LEVEL)

CONCEPT-1

A body feels weightless in projectile motion as it is free-fall motion (always acceleration g downward).

Ex. *A hunter aims his gun and fires a bullet directly towards a monkey sitting on a distant tree. At the instant the bullet leaves the barrel of the gun, the monkey drops from the tree freely. Will the bullet hit the monkey ?*

Sol. Let the horizontal distance to the tree be x and original height of the monkey be H as shown in fig. The angle of projection θ will be given by $\tan \theta = (H/x)$.



If there were no gravity, the bullet would reach height H in the time t taken by it to travel the horizontal distance x.

i.e., $H = (u \sin \theta) \times t$ with $t = (x/u \cos \theta)$

However, because of gravity, the bullet has an acceleration 'g' vertically downwards; so in time t, the bullet will reach a height

$$y = (u \sin \theta) \times t - \left(\frac{1}{2}\right) gt^2 = H - \frac{1}{2} gt^2$$

This is lower than H by $\frac{1}{2}$ gt^2, which is exactly the amount the monkey falls in this time. So the bullet will hit the monkey regardless of the initial velocity of the bullet so long as it is great enough to travel the horizontal distance to the tree before hitting the ground. However, for large u, lesser will be the time of motion; so the monkey is hit near its initial position and for small u, it is hit just before it reaches the ground.

NOTE : Bullet will hit the monkey only and only if

$y > 0$, i.e., $H - \frac{1}{2} gt^2 > 0$

or $H > \frac{1}{2} gt^2$, i.e., $H > \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$

or $u > \frac{x}{\cos\theta} \sqrt{\frac{g}{2H}}$ or $u > \sqrt{\frac{g}{2H}(x^2 + H^2)} = (u_0)$

If $u < u_0$, the bullet will hit the ground before reaching the monkey.

<u>CONCEPT - 2</u>

Particle thrown from top of tower in any direction reaches ground with same velocity, which is same in all cases. (decrease in PE = increase in KE)

Ex. *Three particles A, B and C are thrown from the top of a tower with the same speed. A is thrown straight up, B is thrown straight down and C is thrown horizontally. They hit the ground with speeds υ_A, υ_B and υ_C respectively.*

(A) $\upsilon_A = \upsilon_B = \upsilon_C$ (B) $\upsilon_B > \upsilon_C > \upsilon_A$ (C) $\upsilon_A = \upsilon_B > \upsilon_C$ (D) $\upsilon_A > \upsilon_B = \upsilon_C$

ANS. (A)

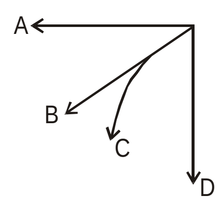
<u>CONCEPT - 3</u>

If air drag acts in vertical direction (opposing motion during ascent and descent) then

$|a_1| > |a_2|$, $t_1 < t_2$, $v_2 < u_1$.

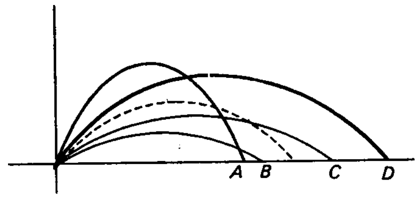
Ex.1 *If wind exerts a constant force in west direction then what is the trajectory of the particle dropped from top of tower (tower is not an obstacle for wind).*

(A) A *(B) B* *(C) C* *(D) D*



ANS. (B)

Ex.2 *The path of a projectile in the absence of air drag is shown in the figure by dotted line. If the air resistance is not ignored then which one of the path shown in the figure is appropriate for the projectile?*



(A) A *(B) B* *(C) C* *(D) D*

ANS. (B)

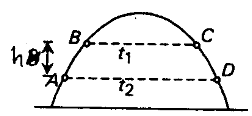
CONCEPT-4

Average speed between two instants when particle is at same level equals to horizontal component of velocity.

CONCEPT - 5

There are two values of time (t_1, t_2) for which h is same. Also $t_1 + t_2 = T$ (time of flight).

In the fig. B and C are at the same level, the time difference between these two positions is t_1 ; A and D are also at the same level, the time difference between these two positions is t_2.



Then, $t_2^2 - t_1^2 = \frac{8h}{g}$

CONCEPT - 6

(a) Horizontal component of velocity (u cosθ), acceleration (g) and mechanical energy remain constant.

(b) Speed, velocity, vertical component of velocity (u sinθ), momentum, kinetic energy and potential energy all change.

(c) Velocity and KE are maximum at the point of projection, while minimum (but not zero) at the highest point.

CONCEPT - 7

PROJECTION FROM A MOVING BODY

(If a particle is projected from some moving body then particle gains the instantaneous velocity only of the moving body and not it's acceleration).

Consider a boy who throws a ball from a moving trolley. Let the velocity of ball relative to boy is u.

$$\vec{V}_{ball, trolley} = \vec{V}_{ball} - \vec{V}_{trolley}$$

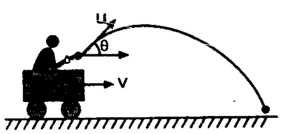
$$\vec{V}_{ball} = \vec{V}_{ball, trolley} + \vec{V}_{trolley}$$

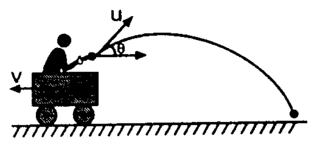
Above equation shows that absolute velocity of ball is vector sum of its velocity with respect to trolley and velocity of trolley. Apply this equation to horizontal as well as vertical motion of the ball. Now consider following cases:

CASE (I) **:** Ball is projected in direction of motion of trolley

Horizontal component of ball's velocity = u cos θ + v

Vertical component of ball's velocity = u sin θ





Horizontal component = u cos θ + v

Vertical component = u sin θ

Horizontal component = u cos θ -v

Vertical component = u sin θ

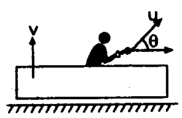
CASE (II) **:** Ball is projected opposite to direction of motion of trolley

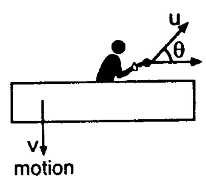
Horizontal component of ball's velocity = u cos θ - v

Vertical component of ball's velocity = u sin θ

CASE (III) : Similarly, for a ball projected upwards from an upward moving platform, horizontal component of ball's velocity = u cos θ

Vertical component of ball's velocity = u sin θ + v





Horizontal component = u cos θ

Vertical component = u sin θ + v

Horizontal component = u cos θ

Vertical component = u sin θ - v

CASE (IV) : For a downward moving platform

Horizontal component of ball's velocity = u cos θ

Vertical component of ball's velocity = u sin θ - v

Ex. *A cart moves with a constant speed along a horizontal circular path. From the cart, a particle is thrown up vertically with respect to the cart.*

(A) The particle will land somewhere on the circular path.

(B) The particle will land outside the circular path.

(C) The particle will follow an elliptical path.

(D) The particle will follow a parabolic path.

ANS. (B), (D)

CONCEPT 8

Initial projection angle so that particle passes through a given point P (x,y): From the equation of trajectory,

$$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha} \quad ....(13)$$

$$y = x \tan \alpha - \frac{gx^2}{2u^2}(1 + \tan^2 \alpha)$$

or $$\tan^2 \alpha - \frac{2u^2}{gx} \tan \alpha + \frac{2u^2 y}{gx^2} + 1 = 0$$

If point P is within the range of projectile then roots of above equation must be real. Complex roots imply that point P is out of range. By method of completing the square,

Putting discriminant of the above equation = 0 ....(14)

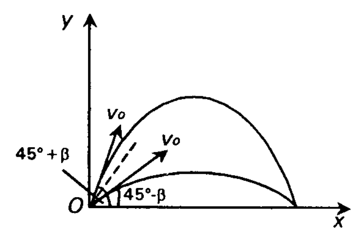
we obtain :

$$\tan \alpha = \frac{u^2}{gx}$$ (where u^2 / gx is the root of the equation) ....(15)

which is the required angle such that the trajectory just reaches point P on the envelope of possible trajectories for a given u.

For complementary angles of projection if T_1 and T_2 are the respective times of flight, then

$$T_1 T_2 = \frac{2R}{g}$$



CONCEPT - 9

COLLISION OF PROJECTILE WITH A WALL.

ASSUMPTION **:** collision is elastic, wall is erect and smooth.

FORMULA USED : speed of approach = speed of separation

Speed of wall does not change after collision too.

NOTATION : v_{xb} = horizontal component of instantaneous speed of projectile just before collision

v_{xa} = horizontal component of instantaneous speed of projectile just after collision in opposite direction.

v_w = instantaneous velocity of wall at the time of collision. Let us take + v_w if wall is coming towards projectile before collision (to avoid complexity, we are not using vector notations here, just applying common sense of collision).

x_1 = horizontal distance covered by projectile before collision.

x_2 = horizontal distance covered by projectile after collision.

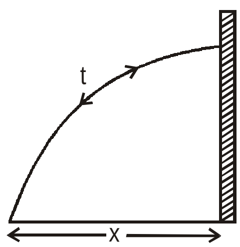
Range = R if wall/collision were absent.

CASE I Wall is stationary

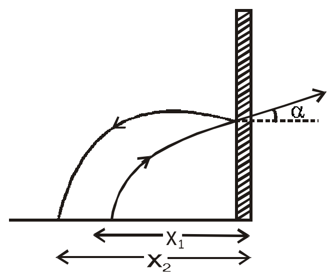
$v_{xa} = v_{xb}$ so $x_1 + x_2$ = Range (if there were no wall).

i.e. ball rebounds with same horizontal speed.

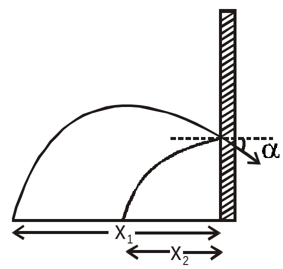
(1)



(2)



(3)



R = 2x

T = 2t

ball was at topmost point at the time of collision (α = 0)

$R = x_1 + x_2$

$T = t_1 + t_2$

ball was moving up at the time of collision

(α = + ve)

$R = x_1 + x_2$

$T = t_1 + t_2$

ball was moving downward at the time of collision (α = - ve)

CASE II Wall is moving with uniform speed v_w towards projectile

speed of approach = speed of separation

$v_{xb} + v_w = v_{xa} - v_w$

$\therefore\ v_{xa} = 2v_w + v_{xb}$

$\therefore$ horizontal component of projectile velocity increases by twice the speed of wall

$\therefore\ x_1 + x_2 >$ Range

CASE III Wall is moving with uniform speed v_w (away from projectile)

speed of approach = speed of separation

$v_{xb} - v_w = v_{xa} + v_w$

$\therefore\ v_{xa} = v_{xb} - 2v_w$

CASE IV Wall is accelerating towards projectile

$\Rightarrow$ same treatment as case II, here we need to find out speed of wall at the time of collision to be used as v_w in formula $v_{xb} + v_w = v_{xa} - v_w$

Hence, on elastic collision from vertical smooth wall (whether at rest or uniform or non-uniform motion, only horizontal component of velocity changes. There is no change in vertical component, so time of flight and maximum height attained remain unchanged.

MISCELLANEOUS EXAMPLES BASED ON OBLIQUE PROJECTILE MOTION

Ex.1 A projectile fired with initial velocity u at some angle θ has a range R. If the initial velocity be doubled at the same angle of projection, then the range will be-

(A) 2R (B) R/2 (C) R (D) 4R

Sol. (D)

$$R = \frac{u^2 \sin 2\theta}{g}$$

$\therefore$ $R \propto u^2$.

If initial velocity be doubled than range will become four times.

Ex.2 If the initial velocity of a projectile be doubled, keeping the angle of projection same, the maximum height reached by it will-

(A) Remain the same (B) Be doubled (C) be quadrupled (D) Be halved

Sol. (C)

$$H = \frac{u^2 \sin^2 \theta}{2g}.$$

$\therefore$ $H \propto u^2$.

If initial velocity be doubled then maximum height reached by the projectile will quadrupled.

Ex.3 In the motion of a projectile freely under gravity, its-

(A) Total energy is conserved

(B) Momentum is conserved

(C) Energy and momentum both are conserved

(D) None is conserved

Sol. (A)

An external force by gravity is present throughout the motion, so momentum will not be conserved.

Ex.4 The range of a projectile for a given initial velocity is maximum when the angle of projection is 45º. The range will be minimum, if the angle of projection is-

(A) 90º (B) 180º (C) 60º (D) 75º

Sol. (A)

$$\text{Range} = \frac{u^2 \sin 2\theta}{g};$$

when θ = 90º, R = 0 i.e. the body will fall at the point of projection after completing one dimensional motion under gravity.

Ex.5 The angle of projection at which the horizontal range and maximum height of projectile are equal is-

(A) 45º (B) $\theta = \tan^{-1}(0.25)$

(C) $\theta = \tan^{-1} 4$ or (θ = 76º) (D) 60º

Sol. (C)

$R = 4H \cot \theta$.

When $R = H$ then $\cot \theta = \frac{1}{4}$

$\Rightarrow \theta = \tan^{-1}(4)$.

Ex.6 At the top of the trajectory of a projectile, the directions of its velocity and acceleration are-

(A) Perpendicular to each other (B) Parallel to each other

(C) Inclined to each other at an angle of 45º (D) Antiparallel to each other

Sol. (A)

Direction of velocity is always tangent to the path so at the top of trajectory, it is in horizontal direction and acceleration due to gravity is always in vertically downward direction. It means $\vec{v}$ and $\vec{g}$ are perpendicular to each other.

Ex.7 An object is thrown along a direction inclined at an angle of 45º with the horizontal direction. The horizontal range of the particle is equal to-

(A) Vertical height (B) Twice the vertical height

(C) Thrice the vertical height (D) Four times the vertical height

Sol. (D)

$R = 4H \cot \theta$

if $\theta = 45º$

then $R = 4H \cot (45º) = 4H$

Ex.8 The height y and the distance x along the horizontal plane of a projectile on a certain planet (with no surrounding atmosphere) are given by $y = (8t - 5t^2)$ meter and $x = 6t$ meter, where t is in second. The velocity with which the projectile is projected is-

(A) 8 m/sec (B) 6 m/sec

(C) 10 m/sec (D) Not obtainable from the data

Sol. (C)

$$v_y = \frac{dy}{dt} = 8 - 10t,$$

$$v_x = \frac{dx}{dt} = 6$$

at the time of projection i.e. [at t = 0], we have, $v_y = \frac{dy}{dt} = 8$ and $v_x = 6$

$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10$ m/s.

Ex.9 Referring to above question, the angle with the horizontal at which the projectile was projected is-

(A) $\tan^{-1}(3/4)$ (B) $\tan^{-1}(4/3)$

(C) $\sin^{-1}(3/4)$ (D) Not obtainable from the given data

Sol. (B)

The angle of projection is given by

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right)$$

Ex.10 A body is projected at such an angle that the horizontal range is three times the greatest height. The angle of projection is :

(A) 25º 8' (B) 33º 7' (C) 42º 8' (D) 53º 8'

Sol. (D)

$$R = \frac{u^2 \sin 2\theta}{g}, \; H = \frac{u^2 \sin^2\theta}{2g}$$

Hence $\frac{H}{R} = \frac{\sin^2\theta}{2\sin 2\theta}$

or $\frac{1}{3} = \frac{1}{4}\tan\theta$

or $\tan\theta = \frac{4}{3}$ or $\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53^\circ\ 8'$.

Ex.11 Range of a projectile is R, when the angle of projection is 30º. Then, the value of the other angle of projection for the same range, is :

(A) 45º (B) 60º (C) 50º (D) 40º

Sol. (B)

Ex.12 A shot is fired from a point at a distance of 200 m from the foot of a tower 100 m high so that it just passes over it. The direction of shot is-

(A) 30º (B) 45º (C) 60º (D) 70º

Sol. (B)

$H_{max.} = 100$ m, $R_{max.} = 2 \times 200 = 400$ m.

$\therefore \tan\theta = \frac{4 \times 100}{400} = 1$ $\left[\because \frac{H}{R} = \frac{\tan\theta}{4}\right]$

$\therefore \theta = 45^\circ$.

PRACTICE QUESTIONS BASED ON OBLIQUE PROJECTILE MOTION

Q.1 For a projectile fired at an angle θ with the horizontal, the maximum height is-
(A) Directly proportional to θ
(B) Directly proportional to $\sin^2 \theta$
(C) Directly proportional to $\sin \theta$
(D) Directly proportional to $\cos \theta$

Q.2 A ball is projected upwards. Its acceleration at the highest point is :
(A) Zero
(B) Directed upwards
(C) Directed downwards
(D) Such as cannot be predicted

Q.3 An arrow is shot into the air. Its range is 200 metres and its time of flight is 5 sec. If the value of g is assumed to be 10 m/sec^2, then the horizontal component of the velocity of arrow is :
(A) 25 m/s (B) 40 m/s (C) 31.25 m/s (D) 12.5 m/s

Q.4 In the Q.3, the vertical component of the velocity is :
(A) 12.5 m/s (B) 31.25 m/s (C) 25 m/s (D) 40 m/s

Q.5 In the Q.3, the maximum height attained by the arrow is :
(A) 12.5 m (B) 25 m (C) 31.25 m (D) 40 m

Q.6 In the Q.3, the angle of projection with the horizontal is :
(A) $\tan^{-1}\left(\frac{4}{5}\right)$ (B) $\tan^{-1}\left(\frac{5}{4}\right)$ (C) $\tan^{-1}\left(\frac{5}{8}\right)$ (D) $\tan^{-1}\left(\frac{8}{5}\right)$

Q.7 The maximum height attained by a projectile is increased by 5%. Keeping the angle of projection constant, what is the percentage increase in horizontal range ?
(A) 5% (B) 10% (C) 15% (D) 20%

Q.8 The maximum height attained by a projectile is increased by 10%. Keeping the angle of projection constant, what is percentage increase in the time of flight ?
(A) 5% (B) 10% (C) 20% (D) 40%

Q.9 If the time of flight of a projectile is doubled, what happens to the maximum height attained?
(A) Halved
(B) Remains unchanged
(C) Doubled
(D) Becomes four times

Q.10 The velocity of projection of a body is increased by 2%. Keeping other factors as constant, what will be percentage change in the maximum height attained ?
(A) 1% (B) 2% (C) 4% (D) 8%

Q.11 In the above question, what will be the percentage change in the time of flight ?
(A) 1% (B) 2% (C) 4% (D) 8%

Q.12 In the above question, what will be the percentage change in the range of projectile ?
(A) 1% (B) 2% (C) 4% (D) 8%

Q.13 A javelin is thrown at an angle θ with the horizontal and the range is maximum. The value of tan θ is-
(A) 1 (B) $\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) 2

Q.14 A stone is thrown at an angle θ with the horizontal such that the horizontal range is equal to the maximum height. The value of tan θ will be-
(A) 1 (B) 2 (C) 3 (D) 4

Q.15 A person can throw a stone to a maximum distance of 100 m. The greatest height to which he can throw the stone is :
(A) 100 m (B) 75 m (C) 50 m (D) 25 m

ANSWERS

1.	(B)	**2.**	(C)	**3.**	(B)	**4.**	(C)	**5.**	(C)	**6.**	(C)
7.	(A)	**8.**	(A)	**9.**	(D)	**10.**	(C)	**11.**	(B)	**12.**	(C)
13.	(A)	**14.**	(D)	**15.**	(C)						

MORE EXAMPLES BASED ON OBLIQUE PROJECTILE MOTION

Ex.1 A body is projected with a velocity of 30 ms^{-1} at an angle of 30^o with the vertical. Find the maximum height, time of flight and the horizontal range.

Sol. Here u = 30 ms^{-1}, Angle of projection, θ = 90 - 30 = 60^o

Maximum height, $H = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{30^2 \sin^2 60^o}{2 \text{ x } 9.8} = 34.44$ m

Time of flight, $T = \dfrac{2u\sin\theta}{g} = \dfrac{2 \text{ x } 30 \sin 60^o}{9.8} = 5.3$s

Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{30^2 \sin 120^o}{9.8} = 79.53$m.

Ex.2 Prove that the maximum horizontal range is four times the maximum height attained by the projectile, when fired at an inclination so as to have maximum horizontal range.

Sol. For $\theta = 45^o$, the horizontal range is maximum and is given by,

$$R_{max} = \frac{u^2}{g}$$

Maximum height attained,

$$H_{max} = \frac{u^2 \sin^2 45^o}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

or $R_{max} = 4H_{max}$

Ex.3 Show that a given cannon will shoot three times as high when elevated at an angle of 60^o as when fired at angle of 30^o but will carry the same distance on a horizontal plane.

Sol. The vertical height attained by a projectile is given by,

$$H = \frac{u^2 \sin^2\theta}{2g}$$

When $\theta = 60^o$, $H_1 = \dfrac{u^2 \sin^2 60^o}{2g} = \dfrac{u^2}{2g}\left(\dfrac{\sqrt{3}}{2}\right)^2 = \dfrac{3u^2}{8g}$

When $\theta = 30^o$, $H_2 = \dfrac{u^2 \sin^2 30^o}{2g} = \dfrac{u^2}{2g}\left(\dfrac{1}{2}\right)^2 = \dfrac{u^2}{8g}$

$\therefore$ $H_1 : H_2 = \dfrac{3u^2}{8g} \times \dfrac{8g}{u^2} = 3 : 1$

Thus the cannon will shoot three times as high when elevated at an angle of 60° as when fired at an angle of 30°.

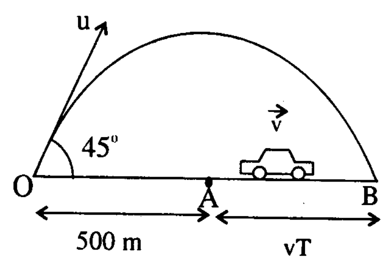
Horizontal range of a projectile, $R = \frac{u^2 \sin 2\theta}{g}$

When $\theta = 60^\circ$, $R_1 = \frac{u^2 \sin 120^\circ}{g} = \frac{\sqrt{3}u^2}{2g}$

When $\theta = 30^\circ$, $R_2 = \frac{u^2 \sin 60^\circ}{g} = \frac{\sqrt{3}u^2}{2g}$

Thus $R_1 = R_2$, i.e. the horizontal distance covered will be same in both cases.

Ex.4 A cannon kept on a straight horizontal road is used to hit a car travelling along the same road away from the cannon with a uniform speed of 72 km/hour. The car is at a distance of 500 m from the cannon when the cannon is fired at an angle of 45º with the horizontal. Find (i) The distance of the car from the cannon when the shell hits it and (ii) the speed of projection of the shell from the cannon.



Sol. Let u be the velocity of projection of the shell. The shell hits the car after T sec at a distance S from the cannon. As the shell will hit the car when the shell reaches the ground. Obviously T is time of flight of shell and S is the range i.e.

$$S = R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 90^\circ}{g} = \frac{u^2}{g}$$

and time of flight

$$T = \frac{2u \sin \theta}{g} = \frac{2u \sin 45^\circ}{g} = \frac{u\sqrt{2}}{g}$$

The shell hits the car at a distance 500 m from the cannon and the velocity of car is constant. Given velocity of car

v = 72 km/hour

$$= \frac{72 \times 1000}{60 \times 60} = 20 \text{ m/s}$$

According to problem

R = 500 + vT

or $\frac{u^2}{g} = 500 + 20 \times \frac{u\sqrt{2}}{g}$

i.e. $u^2 - 20\sqrt{2}\,u - 500g = 0$

or $u^2 - 20\sqrt{2}\,u - 500 \times 9.8 = 0$

Solving, we get

$$u = \frac{20\sqrt{2} \pm \sqrt{[(20\sqrt{2})^2 + 4 \times 500 \times 9.8]}}{2}$$

$$= 10\left[\sqrt{2} \pm \sqrt{51}\right]$$

Taking (+) sign,

$u = 10\,[\sqrt{2} + \sqrt{51}] = 10 \times 8.556 = 85.56$ m/s

(i) $\therefore$ Distance, S = range R

$$= \frac{u^2}{g} = \frac{(85.56)^2}{9.8} = 746.9 \text{ m}$$

(ii) Speed of projection, u = 85.56 m/s.

Ex.5 A ball is thrown from ground level so as to just clear a wall 4m high at a distance of 6m and falls at distance of 14m from the wall. Find the magnitude and direction of the velocity of the ball.

Sol. Let P(x, y) be a point on trajectory of the ball having coordinates (6, 4). As the ball strikes the ground at a distance of 14m from the wall, the range of ball is 6 + 14 = 20m.

The equation of trajectory is

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$= x\tan\theta\left[1 - \frac{gx}{2u^2\cos^2\theta\tan\theta}\right]$$

$$= x\tan\theta\left[1 - \frac{x}{(2u^2\sin\theta\cos\theta/g)}\right]$$

$$= x\tan\theta\left[1 - \frac{x}{R}\right] \qquad \left[\text{Since Range } R = \frac{u^2\sin 2\theta}{g}\right]$$

Substituting x = 6, y = 4 and R = 20, we get

$$4 = 6\tan\theta\left[1 - \frac{6}{20}\right] \text{ or } 4 = 6 \times \frac{14}{20}\tan\theta$$

$$\text{or } \tan\theta = \frac{4 \times 20}{6 \times 14} = \frac{80}{84} = \frac{20}{21}$$

This gives, $\sin\theta = \frac{20}{\sqrt{(841)}}$ and $\cos\theta = \frac{21}{\sqrt{(841)}}$

Now, $R = \frac{u^2\sin 2\theta}{g} = \frac{u^2 .2\sin\theta\cos\theta}{g}$

This gives,

$$u^2 = \frac{Rg}{2\sin\theta\cos\theta} = \frac{20\times 9.8}{2\times\frac{20}{\sqrt{(841)}}\times\frac{21}{\sqrt{(841)}}}$$

$\Rightarrow$ u $\approx$ 14.0 m/s, tan θ = 20/21 = 0.9523 gives θ = 43.6.

Thus, the ball is projected with velocity 14.0 m/s at an angle of projection 43.6º.

Ex.6 A joyful physics student throws his cap into the air with an initial velocity of 24.5 m/s at 36.9º from the horizontal. Find (a) the total time the cap is in the air, and (b) the total horizontal distance travelled.

Sol. PICTURE THE PROBLEM : We choose the origin to be the initial position of the cap, so that $x_{0-} = y_0 = 0$. The total time the cap in the air is found by setting y = 0 in equation. We can then use this result in equation to find the total horizontal distance travelled.

(a) Set y = 0, solve for t :

$$y = v_{0y}\,t - \frac{1}{2}gt^2 = t\,(v_{0y} - \frac{1}{2}gt) = 0$$

There are two solutions for t :

t = 0 (initial conditions)

$$t = \frac{2v_{0y}}{g}$$

Compute the vertical component of the initial velocity vector :

v_{0y} = (24.5 m/s) sin 36.9º = 14.7 m/s

Hence, $t = \frac{2v_{0y}}{g} = \frac{2\times 14.7}{9.8}$ = 3 sec.

Use this value for the time to calculate the total horizontal distance travelled :

$x = v_{0x}\,t = (v_0 \cos\theta)\,t$ = (24.5 m/s) cos 36.9º (3s)

= (19.6 m/s) (3s) = 58.8 m.

Ex.7 A ball is thrown with a velocity of $7\sqrt{2}$ m/s at an angle of 45º with the horizontal. It just clears two vertical poles of height 90 cm each. Find the separation between the poles.

Sol. Let us first calculate the time t after which the ball is at the top of the poles. During this time interval :

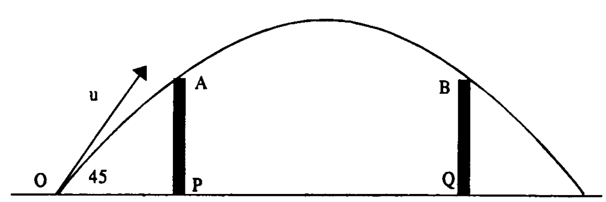
s_y = 0.9 m

u_y = u sin 45° = 7 m/s.

$\Rightarrow$ $s_y = u_y\,t + 1/2\,a_y\,t^2$

$0.9 = 7t - 1/2\,(9.8)\,t^2$

$\Rightarrow$ t = 1/7 or 9/7 s.



Hence, the ball is at A after 1/7 seconds and at B after 9/7 seconds.

OQ = s_x = (u cos 45º) 9/7 = 9m

OP = s_x = (u cos 45º) 1/7 = 1m

$\Rightarrow$ PQ = 8m

ALTERNATIVE METHOD

$y_A = y_B$ = 0.9 m

Using equation of trajectory for y = 0.9 m, we should get the values of x_A and x_B.

$0.9 = x \tan 45^\circ - \dfrac{gx^2}{2(7\sqrt{2})^2 \cos^2 45^\circ}$ has roots x_A and x_B.

On simplification, the equation reduces to;

$x^2 - 10x + 9 = 0$

$\Rightarrow$ $x_A + x_B = 10$ & $x_A x_B = 9$

$\Rightarrow$ $PQ = x_B - x_A = \sqrt{(x_A + x_B)^2 - 4x_A x_B}$

$PQ = 8$ m

Ex.8 A stone is to be thrown so as to cover a horizontal distance of 3m. If the velocity of the projectile is 7 m/s, find :

(i) The angle at which it must be thrown,

(ii) The largest horizontal displacement that is possible with the projection speed of 7 m/s.

Sol. (a) u = 7 m/s, range (R) = 3m

$R = \dfrac{u^2}{g} \sin 2\theta \quad \Rightarrow \quad 3 = \dfrac{7^2}{9.8} \sin 2\theta$

$\Rightarrow$ $\sin 2\theta = 3/5$

$\Rightarrow$ $2\theta = 37^\circ$ or $180^\circ - 2\theta = 37^\circ$

$\Rightarrow$ $\theta = 18^\circ 30'$ or $\theta = 71^\circ 30'$

Hence a range of 3m is possible with two angles of projection

(b) Range is maximum, if $\sin 2\theta$ is maximum.

i.e., for $\sin 2\theta = 1$

$2\theta = 90^\circ$ or $\theta = 45^\circ$

Hence for maximum range with a given velocity, the angle of projection, $\theta = 45^\circ$

$R_{max} = (7^2/9.8) \sin 90^\circ = 5$ m.

<u>NOTE</u>

1. If the angles of projection are complementary ($\alpha + \beta = 90^\circ$) then (for the same projection velocity) the range on the horizontal plane is same.
2. The range on the horizontal plane is maximum for angle of projection $\theta = 45^\circ$. ($R_{max} = u^2/g$)

LEVEL # 1

Q.1 A particle is projected with a velocity v at angle θ with the horizontal. If the mass of the particle is m, then the linear momentum of the particle at the highest point of the trajectory is :

(A) mv (B) mv cos θ (C) mv sin θ (D) $\sqrt{2}$ mv

Q.2 A particle reaches its highest point when it has covered exactly one half of its horizontal range. The corresponding point on the displacement time graph is characterised by-

(A) Negative slope and zero curvature (B) Zero slope and negative curvature
(C) Zero slope and positive curvature (D) Positive slope and zero curvature

Q.3 At the top of the trajectory of a projectile, the acceleration is-

(A) Maximum (B) Minimum (C) Zero (D) g

Q.4 When a body is thrown with a velocity u making an angle θ with the horizontal plane, the maximum distance covered by it in the horizontal direction is-

(A) $\frac{u^2 \sin\theta}{g}$ (B) $\frac{u^2 \sin 2\theta}{2g}$ (C) $\frac{u^2 \sin 2\theta}{g}$ (D) $\frac{u^2 \cos 2\theta}{g}$

Q.5 A football player throws a ball with a velocity of 50 metre/sec at an angle 30 degrees from the horizontal. The ball remains in the air for ($g = 10\ m/s^2$)

(A) 2.5 sec (B) 1.25 sec (C) 5 sec (D) 0.625 sec

Q.6 A body of mass 0.5 kg is projected under gravity with a speed of 98 m/s at an angle of 30º with the horizontal. The change in momentum (in magnitude) of the body is-

(A) 24.5 N-s (B) 49.0 N-s (C) 98.0 N-s (D) 50.0 N-s

Q.7 A gun is aimed at a target in a line of its barrel. The target is released and allowed to fall under gravity. At the same instant, the gun is fired. The bullet will-

(A) Pass above the target (B) Pass below the target
(C) Hit the target (D) Certainly miss the target

Q.8 Two bodies are projected with the same velocity. If one is projected at an angle of 30º and the other at an angle of 60º to the horizontal, the ratio of the maximum heights reached is-

(A) 3 : 1 (B) 1 : 3 (C) 1 : 2 (D) 2 : 1

Q.9 If the range of a gun which fires a shell with muzzle speed V is R, then the angle of elevation of the gun is-

(A) $\cos^{-1}\left(\frac{V^2}{Rg}\right)$ (B) $\cos^{-1}\left(\frac{gR}{V^2}\right)$ (C) $\frac{1}{2}\left(\frac{V^2}{Rg}\right)$ (D) $\frac{1}{2}\sin^{-1}\left(\frac{gR}{V^2}\right)$

PROJECTILE MOTION

Q.10 The time of flight of a projectile is 10 seconds and range is 500 meters. The maximum height attained by it will be-

(A) 125 m (B) 50 m (C) 100 m (D) 150 m

Q.11 A body of mass m is thrown upwards at an angle θ with the horizontal with velocity v. While rising up, the velocity of the mass after t seconds will be-

(A) $\sqrt{(v\cos\theta)^2 + (v\sin\theta)^2}$ (B) $\sqrt{(v\cos\theta - v\sin\theta)^2 - gt}$

(C) $\sqrt{v^2 + g^2t^2 - (2v\sin\theta)gt}$ (D) $\sqrt{v^2 + g^2t^2 - (2v\cos\theta)gt}$

Q.12 A cricketer can throw a ball to a maximum horizontal distance of 100m. With the same effort, he throws the ball vertically upwards. The maximum height attained by the ball is-

(A) 100 m (B) 80 m (C) 60 m (D) 50 m

Q.13 A cricketer can throw a ball to a maximum horizontal distance of 100 m. The speed with which he throws the ball is (to the nearest integer)

(A) 30 ms^{-1} (B) 42 ms^{-1} (C) 32 ms^{-1} (D) 35 ms^{-1}

Q.14 A ball is projected with velocity V_0 at an angle of elevation 30º. Mark the correct statement-

(A) Kinetic energy will be zero at the highest point of the trajectory.

(B) Vertical component of momentum will be conserved.

(C) Horizontal component of momentum will be conserved.

(D) Gravitational potential energy will be minimum at the highest point of the trajectory.

Q.15 Neglecting the air resistance, the time of flight of a projectile is determined by-

(A) $U_{vertical}$ (B) $U_{horizontal}$

(C) $U = [U^2_{vertical} + U^2_{horizontal}]^{1/2}$ (D) $U = [U^2_{vertical} - U^2_{horizontal}]^{1/2}$

Q.16 A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball ? If yes, what should be the angle of projection-

(A) Yes, 60º (B) Yes, 30º (C) No (D) Yes, 45º

Q.17 A stone thrown at an angle θ to the horizontal reaches a maximum height H. Then the time of flight of stone will be-

(A) $\sqrt{\frac{2H}{g}}$ (B) $2\sqrt{\frac{2H}{g}}$ (C) $\frac{2\sqrt{2H\sin\theta}}{g}$ (D) $\frac{\sqrt{2H\sin\theta}}{g}$

Q.18 The horizontal range of a projectile is $4\sqrt{3}$ times its maximum height. Its angle of projection will be-

(A) 45º (B) 60º (C) 90º (D) 30º

PROJECTILE MOTION

Q.19 A ball is projected upwards from the top of tower with a velocity 50 ms^{-1} making an angle 30º with the horizontal. The height of tower is 70m. After how many seconds from the instant of throwing will the ball reach the ground-

(A) 2s (B) 5s (C) 7s (D) 9s

Q.20 Two bodies are thrown up at angles of 45º and 60º respectively with the horizontal. If both bodies attain same vertical height, then the ratio of velocities with which these are thrown is-

(A) $\sqrt{\frac{2}{3}}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\sqrt{\frac{3}{2}}$ (D) $\frac{\sqrt{3}}{2}$

Q.21 At what point of a projectile motion, acceleration and velocity are perpendicular to each other-

(A) At the point of projection

(B) At the point of drop

(C) At the topmost point

(D) Any where in between the point of projection and topmost point.

Q.22 An object is projected at an angle of 45º with the horizontal. The horizontal range and the maximum height reached will be in the ratio-

(A) 1 : 2 (B) 2 : 1 (C) 1 : 4 (D) 4 : 1

Q.23 The maximum horizontal range of a projectile is 400 m. The maximum value of height attained by it will be-

(A) 100 m (B) 200 m (C) 400 m (D) 800 m.

LEVEL # 2

MORE THAN ONE CHOICE MAY BE CORRECT :

Q.1 A grasshopper can jump maximum distance 1.6m. It spends negligible time on the ground. How far can it go in 10 seconds ?

(A) $5\sqrt{2}$ m (B) $10\sqrt{2}$ m (C) $20\sqrt{2}$ m (D) $40\sqrt{2}$ m

Q.2 A body has an initial velocity of 3 m/s and has an acceleration of 1 m/sec^2 normal to the direction of the initial velocity. Then its velocity 4 seconds after the start is :

(A) 7 m/sec along the direction of initial velocity.

(B) 7 m/sec along the normal to the direction of initial velocity.

(C) 7 m/sec mid-way between the two directions

(D) 5 m/sec at an angle of $\tan^{-1}$ (4/3) with the direction of initial velocity.

Q.3 The ceiling of a hall is 40m high. For maximum horizontal distance, the angle at which the ball may be thrown with a speed of 56 ms^{-1} without hitting the ceiling of the hall is :

(A) 25º (B) 30º (C) 45º (D) 60º

Q.4 A shell fired from the ground is just able to cross (in a horizontal direction) the top of a wall 90 m away and 45 m high. The direction of projection of the shell will be-

(A) 25º (B) 30º (C) 60º (D) 45º

Q.5 In the previous question, the velocity of the shell will be :

(A) 42 ms^{-1} (B) 52 ms^{-1} (C) 32 ms^{-1} (D) 62 ms^{-1}

Q.6 If T be the total time of flight of a current of water and H be the maximum height attained by it from the point of projection, then H/T will be- (u = projection velocity, θ = projection angle)

(A) (1/2) u sinθ (B) (1/4) u sinθ (C) u sinθ (D) 2 u sinθ

Q.7 A hunter aims his gun and fires a bullet directly at a monkey on a tree. At the instant bullet leaves the gun, monkey drops, the bullet:

(A) hits the monkey (B) misses to hit the monkey

(C) can not be said (D) None of these

Q.8 A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the times of flight in two cases, then the product of times of flight will be-

(A) $t_1t_2 \propto R$ (B) $t_1t_2 \propto R^2$ (C) $t_1t_2 \propto 1/R$ (D) $t_1t_2 \propto 1/R^2$

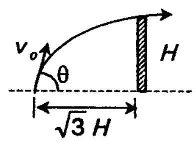
Q.9 What θ should be used such that projectile crosses a wall of height h and drops in a dig on the ground h metres away from wall and 3n metres away from man throwing projectile.

(A) $\tan^{-1}5/2$ (B) $\tan^{-1}3/2$ (C) $\tan^{-1}7/2$ (D) $\tan^{-1}1/2$

Q.10 The equation of projectile is $y = \sqrt{3}x - \frac{1}{2}gx^2$. The velocity of projection is-

(A) 1 m/s (B) 2 m/s (C) 3 m/s (D) 4 m/s

Q.11 A projectile is thrown at an angle θ such that it is just able to cross a vertical wall as shown in the figure.



The angle θ at which the projectile is thrown is given by

(A) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\tan^{-1}\sqrt{3}$ (C) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (D) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Q.12 If a base ball player can throw a ball at maximum distance = d over a ground, the maximum vertical height to which he can throw it, will be- (Ball has same initial velocity in each case)

(A) d/2 (B) d (C) 2d (D) d/4

Q.13 What is the average velocity of a projectile between the instants it crosses half the maximum height? (it is projected with a speed u at an angle θ with the horizontal)

(A) u sinθ (B) u cosθ (C) u tanθ (D) u

Q.14 A boy throws a ball with a velocity V_o at an angle α to the horizontal. At the same instant he starts running with uniform velocity to catch the ball before it hits the ground. To achieve this, he should run with a velocity of-

(A) $V_o \cos\alpha$ (B) $V_o \sin\alpha$ (C) $V_o \tan\alpha$ (D) $\sqrt{V_0^2 \tan\alpha}$

Q.15 Bullet and target (stone) are at same level. When bullet is fired, stone is dropped simultaneously. Will bullet hit the stone?

(A) yes (B) no (C) can't say

Q.16 A ball is projected with velocity v_0 at an angle θ with the ground. The time after which the velocity of the ball is perpendicular to its initial direction of motion is

(A) $\frac{v_0}{g\cos\theta}$ (B) $\frac{v_0}{g\sin\theta}$ (C) $\frac{v_0}{g}\tan\theta$ (D) $\frac{v_0}{g}\cot\theta$

Q.17 A ball is projected with velocity v_0 at an angle θ with the ground. The velocity of the ball at the instant when its velocity is perpendicular to its initial direction of motion is-

(A) $\frac{v_0}{\cos\theta}$ (B) $\frac{v_0}{\sin\theta}$ (C) $v_0 \tan\theta$ (D) $v_0 \cot\theta$

PROJECTILE MOTION

Q.18 An artillery piece which consistently shoots its shell with the same muzzle speed has a maximum range of R. To hit a target which is R/2 from the gun and on the same level, at what elevation angle should the gun be pointed -

(A) 30° (B) 45° (C) 60° (D) 75°

Q.19 A projectile is thrown at an angle of 45º with the ground. At the highest position, the radius of curvature of the projectile is-

(A) Equal to its maximum height (B) Equal to two times its maximum height

(C) Equal to half of its horizontal range (D) Equal to its horizontal range

Q.20 A large number of bullets are fired in all directions with the same speed v. What is the maximum area on the ground on which these bullets will spread-

(A) $\frac{\pi v^2}{g}$ (B) $\frac{\pi v^4}{g^2}$ (C) $\frac{\pi^2 v^4}{g^2}$ (D) $\frac{\pi^2 v^2}{g^2}$

Q.21 The height y and the distance x along the horizontal plane of a projectile on a certain planet (with no surrounding atmosphere) are given by $y = (8t - 5t^2)$ meter and $x = 6t$ meter where t is time in seconds. The velocity with which the projectile is projected is -

(A) 8 m/s (B) 6 m/s

(C) 10 m/s (D) Cannot be determined

Q.22 A shot is fired from a gun with a muzzle velocity of 98 m/s at an angle of elevation 45º from the ground. Its range is found to be 900 m. The range decreased by air resistance is

(A) 8 m (B) 80 m (C) 98 m (C) 9.8 m

Q.23 The muzzle velocity of a shell is 280 m/s. At what angle must the gun be fired if the projectile is to strike a target at the same level as the gun at distance 4 km ?

(A) 10º (B) 30º (C) 15º (D) 45º

Q.24 If the maximum horizontal range for a projectile is R, the greatest height attained by it is

(A) 4R (B) R/2 (C) 2R (D) R/4

Q.25 The velocity at the maximum height of a projectile is half of its initial velocity u. Its range on the horizontal plane is

(A) $\frac{2u^2}{3g}$ (B) $\frac{3u^2}{g}$ (C) $\frac{\sqrt{3}u^2}{g}$ (D) $\frac{u^2}{3g}$

Q.26 The velocity of projection of a particle if it does not rise more than 3 m in a range of 600 m is

(A) 383.4 m/s (B) 273 m/s (C) 343 m/s (D) 3.83 m/s

Q.27 A cannon ball has the same range R on a horizontal plane for two angles of projection. If h_1 and h_2 are the greatest heights in the two paths for which this is possible, then

(A) $R = h_1h_2$ (B) $R = 4\sqrt{h_1h_2}$ (C) $R = 3\sqrt{h_1h_2}$ (D) $R = (h_1h_2)^{1/4}$

PROJECTILE MOTION

Q.28 A particle of mass m is projected with velocity v making an angle of 45º with the horizontal. The magnitude of the angular momentum of the particle about the point of projection when the particle is at its maximum height is (where g = acceleration due to gravity)

(A) Zero (B) $\frac{mv^2}{2g}$ (C) $\frac{mv^3}{(\sqrt{2}g)}$ (D) $\frac{mv^3}{(4\sqrt{2}g)}$

Q.29 If the range of the projectile be R, then the potential energy will be maximum after the projectile has covered (from start) a distance equal to-

(A) R (B) R/2 (C) R/4 (D) R/8

Q.30 A projectile is thrown into space so as to have maximum possible horizontal range equal to 400 m. Taking the point of projection as the origin, the coordinates of the point where the velocity of projectile is minimum are-

(A) (400, 100) (B) (200, 100) (C) (400, 200) (D) (200, 200)

Q.31 A projectile is projected with a linear momentum p making angle θ with the horizontal. The change in momentum of the projectile on return to the ground will be-

(A) 2p (B) 2p cos θ (C) 2p sin θ (D) 2p tan θ.

Q.32 Two balls A and B are thrown with speeds u and u/2 respectively. Both the balls cover the same horizontal distance before returning to the plane of projection. If the angle of projection of ball B is 15^{o} with the horizontal, then the angle of projection of A is-

(A) $\sin^{-1}$ (1/8) (B) (1/2) $\sin^{-1}$ (1/8) (C) (1/3) $\sin^{-1}$ (1/8) (D) (1/4) $\sin^{-1}$ (1/8)

Q.33 A particle is projected at an angle of elevation α and after t seconds it appears to have an angle of elevation β as seen from point of projection. The initial velocity will be-

(A) $\frac{gt}{2\sin(\alpha-\beta)}$ (B) $\frac{gt\cos\beta}{2\sin(\alpha-\beta)}$ (C) $\frac{\sin(\alpha-\beta)}{2gt}$ (D) $\frac{2\sin(\alpha-\beta)}{gt\cos\beta}$

Q.34 A cannon ball has a range R on a horizontal plane. If h and h' are the greatest heights in the two paths for which this is possible, then-

(A) $R = 4\sqrt{(hh')}$ (B) $R = \frac{4h}{h'}$ (C) R = 4 hh' (D) $R = \sqrt{(hh')}$

Q.35 A projectile fired at an angle θ with the vertical experiences a force due to air resistance. The resistive force is directly proportional to the instantaneous speed of the projectile. Which one of the following statements is correct-

(A) The path of the projectile is a parabola symmetric about a vertical line passing through the highest point

(B) At the highest point, speed is horizontal

(C) The time of ascent is 1.5 times the time of descent as it would be in the absence of air resistance.

(D) None of these.

Q.36 A ball rolls off the top of a stair way with a horizontal velocity u m/s. If the steps are h metres high and b metres wide, the ball will hit the edge of the nth step, if-

(A) $n = 2hu/gb^2$ (B) $n = 2hu^2 / gb^2$ (C) $n = 2hu^2 / gb$ (D) $n = hu^2 / gb^2$

Q.37 Three particle A, B and C are projected from the same point with same initial speeds making angles 30º, 45º and 60º respectively with the horizontal. Which of the following statements is correct ?

(A) A, B and C have unequal ranges

(B) Ranges of A and C are equal and less than that of B

(C) Ranges of A and C are equal and greater than that of B

(D) A, B and C have equal ranges.

Q.38 A body is thrown with a velocity of 9.8 m/s making an angle of 30º with the horizontal. It will hit the ground after a time.

(A) 3s (B) 2s (C) 1.5 s (D) 1s

Q.39 A javelin thrown into air at an angle with the horizontal has a range of 200 m. If the time of flight is 5 second, then the horizontal component of velocity of the projectile at the highest point of the trajectory is-

(A) 40 m/s (B) 0 m/s (C) 9.8 m/s (D) Infinite

Q.40 If a particle follows the trajectory $y = x - \frac{1}{2}x^2$, then the time of flight is :

(A) $\frac{1}{\sqrt{g}}$ (B) $\frac{2}{\sqrt{g}}$ (C) $\frac{3}{\sqrt{g}}$ (D) $\frac{4}{\sqrt{g}}$

Q.41 An object is projected at an angle to the horizontal in a gravitational field and it follows a parabolic path, PQRST. These points are the positions of the object after successive equal time intervals, T being the highest point reached. The displacements PQ, QR and ST

(A) are equal (B) decrease at a constant rate.

(C) have equal horizontal components. (D) increase at a constant rate.

Q.42 A ball of mass m is projected from the ground with an initial velocity u making an angle θ with the horizontal. What is the change in velocity between the point of projection and the highest point

(A) $u\cos^2\theta$ downward (B) $u\cos\theta$ upward

(C) $u\sin^2\theta$ upward (D) $u\sin\theta$ downward

Q.43 At a certain moment of time, the angle between velocity vector v and the acceleration a of a particle, is greater than 90^o. What can be inferred about its motion at that moment?

(A) It is curvilinear and decelerated (B) It is rectilinear and accelerated

(C) It is curvilinear and accelerated (D) It is rectilinear and decelerated

PROJECTILE MOTION

Q.44 A projectile is thrown into air from a point on the horizontal ground at an angle with the horizontal. If the air exerts a resistive force proportional to the speed of the projectile

(A) The path of the projectile will be symmetric about the vertical line through its highest position

(B) At the highest point, the velocity is horizontal

(C) The time for ascent equals the time for descent

(D) The time for ascent is less than that for descent.

Q.45 A particle moves in the XY plane according to the law $x = a\sin\omega t$ and $y = a(1 - \cos\omega t)$, where a and ω are constants. Then, the trajectory of the particle is

(A) a parabola

(B) a straight line equally inclined at x and y-axes

(C) a circle

(D) an ellipse.

Q.46 A particle is thrown with a speed u at an angle θ with the horizontal. When the particle makes an angle ϕ with the horizontal, its speed changes to υ.

(A) $\upsilon = u\cos\theta$

(B) $\upsilon = u\cos\theta . \cos\phi$

(C) $\upsilon = u\cos\theta . \sec\phi$

(D) $\upsilon = u\sec\theta . \cos\phi$

ANSWER KEY

LEVEL # 1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	B	D	C	C	B	C	B	D	A	C	D	C	C	A
Que.	16	17	18	19	20	21	22	23							
Ans.	A	B	D	C	C	C	D	B							

LEVEL # 2

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	D	B	D	A	B	A	A	B	B	C	A	B	A	A
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	D	D	B,C	B	C	B	C	D	C	A	B	D	B	B
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	C	B	B	A	B	B	B	D	A	B	C	D	A	B,D	C
Que.	46														
Ans.	C														