MOTION IN TWO DIMENSIONS

LAW OF INDEPENDENCE OF DIRECTION

In general, the motion of a particle may be resolved simultaneously along the three mutually perpendicular directions, i.e. x, y and z-axes. According to the law of independence of directions, the motion of a particle along the x-axis is completely independent of it's motion along the y and z-axis.

We can always divide one 2-D motion into two 1-D motions and use equations of motion separately.

Components of $\vec{v}$, $\Delta\vec{r}$, $\vec{a}$ which are perpendicular to each other are always independent of each other. So x, v_x, a_x are independent of y, v_y, a_y.

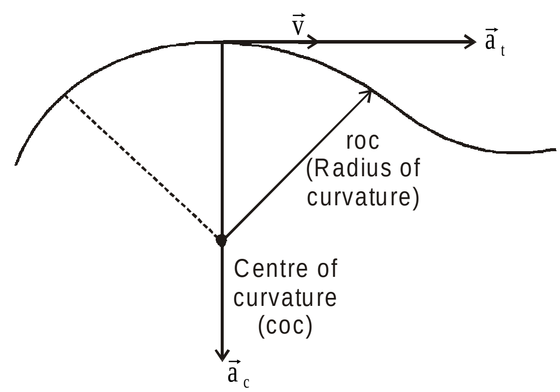
TRAJECTORY

y = f (x)

RADIUS OF CURVATURE/CENTER OF CURVATURE

y = f (x)

$$R_c = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$



TANGENTIAL AND RADIAL (NORMAL) ACCELERATIONS

Ex.1 *If the coordinates of position of a particle are given as x = A sin ωt , y = A (1-cos ωt) then find*

(a) angle between $\vec{v}$ and $\vec{a}$ (b) distance covered in time T.

Sol. (a) $v_x = \frac{dx}{dt} = A\omega \cos \omega t$, $v_y = \frac{dy}{dt} = A\omega \sin \omega t$

$\therefore$ $|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{(A\omega\cos\omega t)^2 + (A\omega\sin\omega t)^2} = A\omega$

$\vec{v} = v_x\hat{i} + v_y\hat{j} = A\omega \cos\omega t\ \hat{i} + A\omega \sin\omega t\ \hat{j}$

$a_x = \frac{dv_x}{dt} = -A\omega^2 \sin\omega t$, $a_y = \frac{dv_y}{dt} = A\omega^2 \cos\omega t$

$\therefore$ $\vec{a} = a_x\hat{i} + a_y\hat{j} = -A\omega^2 \sin\omega t\ \hat{i} + A\omega^2 \cos\omega t\ \hat{j}$

now as $\cos\theta = \dfrac{\vec{a}.\vec{v}}{|a||v|}$

so $\cos\theta = -\sin(4\pi) = 0$ (at t = T)

$\therefore$ $\theta = \pi/2$ ANS.

(b) Distance covered in T = |v| x T = $A\omega T$ ANS.

***Ex.2** If $a_x = 2t$ and $a_y = 1$ then find equation of trajectory if s = 0 and v = 0 at t = 0*

Sol. $\int_0^{v_x} dv_x = \int_0^t a_x\,dt = \int 2t\,dt = t^2$

$\therefore$ $v_x = t^2$

$\int_0^x dx = \int_0^t v_x\,dt = \int t^2\,dt = t^3/3$

$\therefore$ $x = \dfrac{t^3}{3}$ $\therefore$ $t = (3x)^{1/3}$ ....(i)

Similarly $y = \dfrac{t^2}{2}$ $\therefore$ $t = (2y)^{1/2}$ ....(ii)

from (i) and (ii), $(3x)^{1/3} = (2y)^{1/2}$ or $(3x)^2 = (2y)^3$ or $9x^2 = 8y^3$ ANS.

***Ex.3** A particle is moving in x - y plane with $v = a\hat{i} + bx\hat{j}$. Find (a) equation of trajectory (b) radius of curvature of trajectory as f (x). Assume initial conditions to be zero.*

Sol. (a) $v_x = a$ $\therefore$ $x = at$ so $t = x/a$ ...(i)

$\dfrac{dy}{dt} = v_y = bx$ $\therefore$ $\int dy = \int bx\,dt = \int bat\,dt = ab\dfrac{t^2}{2}$, so $t = \left[\dfrac{2y}{ab}\right]^{1/2}$ ...(ii)

$\therefore$ from (i), (ii) : $\dfrac{x}{a} = \left(\dfrac{2y}{ab}\right)^{1/2}$

$\therefore$ $y = \dfrac{bx^2}{2a}$ ANS.

(b) $\dot{y} = \dfrac{dy}{dx} = \dfrac{b}{a}x$, $\ddot{y} = \dfrac{d^2y}{dx^2} = \dfrac{b}{a}$

so $R_c = \dfrac{\left[1+(\dot{y})^2\right]^{3/2}}{\ddot{y}} = \dfrac{\left[1+\dfrac{b^2x^2}{a^2}\right]^{3/2}}{b/a} = \dfrac{a}{b}\left[1+\dfrac{b^2x^2}{a^2}\right]^{3/2}$ ANS.

Ex.4 *Positions of a particle are as $x = at$, $y = at(1 - \alpha t)$. Find time at which angle between $\vec{v}$ and $\vec{a}$ equals to $\pi/4$.*

Ans. ($t = 1/\alpha$) (Solve yourself)

Ex.5 *A balloon is flowing in the air with velocity vectors as $v_x = by$ and $v_y = v_o$. Find trajectory equation and tangential and normal accelerations.*

Hint : $x = \frac{by^2}{2v_o}$, $a_t = a_x \cos\theta = bv_o \cos\theta = \frac{b^2 v_o y}{\sqrt{v_o^2 + b^2 y^2}}$

$$a_n = a_x \sin\theta = \frac{bv_o^2}{\sqrt{v_o^2 + b^2 y^2}}$$

NOTE

Rest of the examples of projectile motion and circular motion will be discussed in coming chapters.