CIRCULAR MOTION

DEFINITION : If the trajectory of a moving particle is along a circle i.e. $(x - h)^2 + (y - k)^2 = r^2$, then motion is called circular motion. Velocity vector in circular or any curved path is always along tangent to trajectory.

TYPES OF CIRCULAR MOTION

(A) UNIFORM CIRCULAR MOTION

(B) NON UNIFORM CIRCULAR MOTION

UNIFORM CIRCULAR MOTION

If the speed of the particle is constant with time, then motion is called uniform circular motion.

NOTE : Here |velocity| is constant, but the velocity vector is changing due to continuous change in direction, so here acceleration is there due to change of $\vec{v}$ (direction only).

PARAMETERS DEFINING CIRCULAR MOTION

All the parameters defined so far for linear motion like displacement, velocity and acceleration are equivalently defined for circular motion also with prefix "angular". Let's have a look at them one by one with a specific sign convention.

SIGN CONVENTION

Anticlockwise parameters $\Rightarrow$ positive

Clockwise parameters $\Rightarrow$ negative

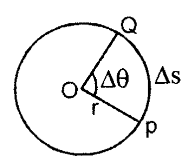
ANGULAR DISPLACEMENT

Angle subtended by position vector of a particle moving along any arbitrary path w.r.t. some fixed point is called angular displacement.

(i) Angular displacement is an axial vector, if small and not a vector, if large as commutative property for vector addition does not hold true.

i.e. $d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1$

but $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$



(ii) It's direction is perpendicular to plane of rotation and is given by right hand screw rule. i.e. if body is moving in (x - y) plane in anticlockwise manner, then $|\Delta\theta|$ is +ve and direction is along +z axis.

NOTE : Clockwise angular displacement is taken as negative and anticlockwise displacement as positive.

$$\text{angle} = \frac{\text{arc}}{\text{radius}} = \frac{\text{linear displacement}}{\text{radius}}$$

(iii) For circular motion : $\Delta s = r \times \Delta\theta$

(iv) Its unit is radian (in M.K.S.)

NOTE : Always change degree into radian, if it occurs in numerical problems.

NOTE : 1 radian = $\frac{360^o}{2\pi}$ and π radian = 180^o

(v) If a body makes n revolutions, it's angular displacement, $\theta = 2n\pi$ radians

(vi) It is a dimensionless quantity i.e. dimension = $[M^oL^oT^o]$

Ex. *A particle completes 1.5 revolutions in a circular path of radius 2cm. The angular displacement of the particle will be - (in radian)*

(A) π *(B) 2π* *(C) 3π* *(D) 4π*

Sol. (C)

We have, angular displacement = $\frac{\text{linear displacement}}{\text{radius of path}}$

$\Rightarrow \quad \Delta\theta = \frac{\Delta S}{r}$

Here, $\Delta S = n(2\pi r)$

$= 1.5\ (2\pi \times 2 \times 10^{-2}) = 6\pi \times 10^{-2}$

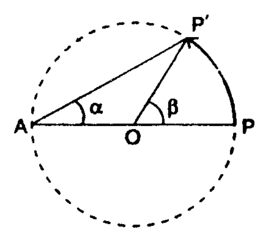
$\therefore \quad \Delta\theta = \frac{6\pi \times 10^{-2}}{2 \times 10^{-2}} = 3\pi$ radian

ANGULAR VELOCITY

DEFINITION

The rate of change of angular position of a body moving in a circular motion is called angular velocity.

$$\omega = \frac{d\theta}{dt}$$



Angular velocity depends on the point about which rotation is considered e.g. if a particle is moving in a circle from p to p' in time t as shown in Fig, the angular velocity with respect to O will be $\omega_o = (\beta/t)$, while with respect to A will be $\omega_A = (\alpha/t)$.

But, from geometry of figure, $\beta = 2\alpha$ so, $\omega_o = 2\omega_A$.

TYPES

(I) AVERAGE ANGULAR VELOCITY

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

Ex.1 *A particle revolving in a circular path completes first one third of circumference in 2sec, while next one third in 1sec. The average angular velocity of particle will be (in rad/sec)*

(A) $\frac{2\pi}{3}$ *(B)* $\frac{4\pi}{3}$ *(C)* $\frac{2\pi}{9}$ *(D)* $\frac{4\pi}{9}$

Sol. (D)

We have $\vec{\omega}_{av} = \frac{\text{Total angular displacement}}{\text{Total time}}$

For first one third part of circle, angular displacement, $\theta_1 = \frac{S_1}{r} = \frac{2\pi r/3}{r} = \frac{2\pi}{3}$ radian

For second one third part of circle, $\theta_2 = \frac{2\pi r/3}{r} = \frac{2\pi}{3}$ rad.

Total angular displacement, $\theta = \theta_1 + \theta_2 = \frac{2\pi}{3} + \frac{2\pi}{3} = \frac{4\pi}{3}$ rad

Total time = 2 + 1 = 3 sec.

$\therefore\ \vec{\omega}_{av} = \frac{4\pi/3}{3}$ rad/s $= \frac{4\pi}{9}$ rad/s

Ex.2 *The ratio of angular speeds of minute hand and hour hand of a watch is -*

(A) 3 : 1 *(B) 4 : 1* *(C) 12 : 1* *(D) 16 : 1*

Sol. (C)

Angular speed of hour hand, $\omega_1 = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{12 \times 60 \times 60}$ rad/sec

Angular speed of minute hand, $\omega_2 = \frac{2\pi}{60 \times 60}$ rad/sec $\Rightarrow \frac{\omega_2}{\omega_1} = \frac{12}{1}$

(II) INSTANTANEOUS ANGULAR VELOCITY

$$\omega_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Ex. *The angular displacement of a particle is given by* $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, *where* ω_0 *and* α *are constant and* ω_0 *= 1 rad/sec,* α *= 1.5 rad/sec². The angular velocity at time t = 2 sec will be (in rad/sec)*

(A) 2 rad/sec *(B) 3 rad/sec* *(C) 4 rad/sec* *(D) 5 rad/sec*

Sol. (C)

We have $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \quad \Rightarrow \frac{d\theta}{dt} = \omega_0 + \alpha t$

This is angular velocity at time t.
Now angular velocity at t = 2 sec will be

$$\omega = \left(\frac{d\theta}{dt}\right)_{t=2sec} = \omega_0 + 2\alpha = 1 + 2 \times 1.5 = 4 \text{ rad/sec.}$$

(III) UNIFORM ANGULAR VELOCITY : Particle is covering equal angle in equal interval of time, so speed of particle is constant.

(IV) NON-UNIFORM ANGULAR VELOCITY : Speed of particle is varying with time.

ANGULAR ACCELERATION

DEFINITION

The rate of change of angular velocity is called angular acceleration.

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

TYPES

(I) AVERAGE ANGULAR ACCELERATION

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$

Ex. *A particle travels in a circle of radius 20 cm at a speed that increases uniformly. If the speed changes from 5 m/s to 6m/s in 2s, then angular acceleration is-*

(A) 1.5 rad/sec² *(B) 2.0 rad/sec²* *(C) 2.5 rad/sec²* *(D) 3.0 rad/sec²*

Sol. (C)

$$a_t = \frac{d|v|}{dt} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{6-5}{2} = 0.5 \text{ m/s}^2$$

$$\therefore \quad \alpha = \frac{a_t}{r} = \frac{0.5\,m/s^2}{20\,cm} = \frac{0.5\,m/s^2}{0.2\,m} = 2.5 \text{ rad/s}^2$$

(II) INSTANTANEOUS ANGULAR ACCELERATION

$$\alpha_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Ex. *The angular velocity of a particle moving in a circular path is given by :*

$\omega = 2t^2 + 3t$

(Where ω is a function of time). Now, the instantaneous angular acceleration at t = 2 sec is given by :

(A) 5 rad/sec² *(B) 7 rad/sec²* *(C) 9 rad/sec²* *(D) 11 rad/sec²*

Sol. (D)

Here, $\omega = 2t^2 + 3t$

$\therefore$ Instantaneous angular acceleration $= \alpha = \frac{d\omega}{dt}$

or $\frac{d\omega}{dt} = (4t + 3)$

$\therefore$ $\left(\frac{d\omega}{dt}\right)_{at\ t=2} = 4 \times 2 + 3 = 11$ rad/sec^2.

(III) UNIFORM ANGULAR ACCELERATION : The magnitude of acceleration is constant.

(IV) NON-UNIFORM ANGULAR ACCELERATION : The magnitude of acceleration is varying with time.

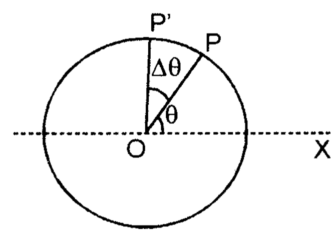
DERIVATION OF $v = \omega r$ ($\vec{v} = \vec{\omega} \times \vec{r}$) AND $a = \alpha r \Rightarrow \vec{a} = \vec{\alpha} \times \vec{r}$

Let Particle P moves in a circle

Where, r = radius of circle.

O be the centre of circle

Let, O be origin & OX = x - axis.



Angular Position of particle = θ

As particle moves, θ changes.

In time Δt, particle moves $\Delta\theta$.

$\therefore$ Angular velocity $\Rightarrow \omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

Now, Angular acceleration = Rate of change of Angular velocity

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$\therefore$ linear distance PP' travelled by particle in time

$\Delta t \Rightarrow \Delta s = r\Delta\theta$

or $\frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$

or $v = r\omega$ ....(i)

Where, v = linear speed of particle

<u>IN VECTOR FORM</u> ($\vec{v} = \vec{\omega} \times \vec{r}$)

If $\vec{\omega} = \omega_o\hat{k}$, $\vec{r} = r_o\hat{i}$, then $\vec{v} = \omega_o r_o\hat{j}$ (So anticlockwise rotation)

Differentiating (i)

$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt}$ or $a_t = r\alpha$

IN VECTOR FORM $(\vec{a} = \vec{\alpha} \times \vec{r})$

$\therefore$ a_t = Rate of change of speed.

Ex.1 *The angular velocity of a particle moving with a linear velocity of 5m/sec in a circle of radius 2m is given as :*

(A) 2.5 rad/sec *(B) 3 rad/sec* *(C) 4 rad/sec* *(D) 5 rad/sec*

Sol. (A)

Linear velocity = v = 5 m/sec

Radius = 2m

$\therefore$ Angular velocity = $\omega = \dfrac{\text{Linear velocity}}{\text{Radius}}$

$= \dfrac{v}{r} = \dfrac{5}{2}$ rad/sec = 2.5 rad/sec.

Ex.2 *Two cars A & B are going around concentric circular paths of radii r_A & r_B. If the two cars complete the circular paths in the same time, then the ratio of angular speeds of A & B is:*

(A) 1 *(B) $\dfrac{r_A}{r_B}$* *(C) $\dfrac{r_B}{r_A}$* *(D) Indeterminate*

Sol. (A)

Cars complete the circles in equal time.

Means that angular displacement of both cars are equal in equal time.

Means angular velocities are equal too.

Hence, $\dfrac{\omega_A}{\omega_B} = 1$.

Ex.3 *The angular acceleration of a particle on circular path of radius 2m is 5 rad/s^2. The rate of increase of speed of the particle is -*

(A) 2.5 m/s^2 *(B) 3 m/s^2* *(C) 7 m/s^2* *(D) 10 m/s^2*

Sol. (D)

Rate of increase of speed = linear acceleration

$a_t = r\alpha = 2 \times 5 = 10$ m/s^2.

EQUATIONS OF CIRCULAR MOTION FOR UNIFORM ANGULAR ACCELERATION

1. We know that, $\dfrac{d\omega}{dt} = \alpha$ [Where α = constant angular acceleration]

or $d\omega = \alpha\, dt$

integrating both sides, we get

$$\int_{\omega_o}^{\omega} d\omega = \int_o^t \alpha\, dt \quad \text{or} \quad \omega - \omega_o = \alpha t$$

$\Rightarrow$ $\omega = \omega_o + \alpha t$ .... (i)

Where, ω_o = initial angular velocity at time t = 0 ; ω = Final angular velocity at time t = t

Ex. *A particle starts from rest with an angular acceleration of 2 rad/s² in a circle of radius 2m. Find it's linear speed after 6 seconds-*

(A) 12 ms^{-1} *(B) 24 ms^{-1}* *(C) 4 ms^{-1}* *(D) None of these*

Sol. (B)

Here $\omega_0 = 0,\ \alpha = 2$ rad/sec²

applying $\omega = \omega_0 + \alpha t = 0 + 2(6) = 12$ rad sec^{-1}

$v = r\omega = 24\ ms^{-1}$.

2. Again we can write $\omega = \dfrac{d\theta}{dt}$.

$\therefore$ From equation (i), we get

$$\frac{d\theta}{dt} = \omega_o + \alpha t$$

or $d\theta = (\omega_0 + \alpha t)\, dt$

On integrating,

$$\int_o^\theta d\theta = \int_o^t (\omega_o + \alpha t)\, dt$$

or $\theta = \omega_o t + \dfrac{1}{2}\alpha t^2$. ....(ii)

where, ω_o = initial angular velocity at time t = 0 ; θ = Final angular position

Ex. *A particle, starting from rest went through an angular displacement of $\left(\dfrac{2\pi}{3}\right)$ radians, while moving with angular acceleration of π rad/s². The time taken in the process is :*

(A) $\dfrac{1}{\sqrt{3}}$ s *(B) $\dfrac{2}{\sqrt{3}}$ s* *(C) $\sqrt{3}$ s* *(D) $2\sqrt{3}$ s*

Sol. (B)

$$s = \omega_0 t + \frac{1}{2}\alpha t^2; \quad s = \frac{2\pi}{3}\text{rad},\ \omega_0 = 0,\ \alpha = \pi \text{ rad/sec}^2$$

So $\dfrac{2\pi}{3} = \dfrac{1}{2} \times \pi \times (t)^2$

$\Rightarrow$ $t^2 = \dfrac{4}{3}$ $\Rightarrow$ $t = \sqrt{\dfrac{4}{3}}$ sec $= \dfrac{2}{\sqrt{3}}$ sec.

3. Again $\dfrac{d\omega}{dt} = \alpha$ or $\dfrac{d\omega}{d\theta}\cdot\dfrac{d\theta}{dt} = \alpha$

$\because$ $\dfrac{d\theta}{dt} = \omega$ $\therefore$ $\dfrac{d\omega}{d\theta}\cdot\omega = \alpha$

$\Rightarrow \omega\, d\omega = \alpha\, d\theta$.

On integrating

$$\int_{\omega_o}^{\omega} \omega d\omega = \int_{o}^{\theta} \alpha d\theta$$

or $\omega^2 - \omega_0^2 = 2\alpha\theta$

or $\omega^2 - \omega_0^2 - 2\alpha\theta = 0$ ....(iii)

Equations (i), (ii) and (iii) can be used to find different parameters of a particle which is moving with constant angular acceleration.

Ex. *A particle moving with 3 rad/sec is accelerated upto 7 rad/sec at the rate of 2 rad/s^2. The angular displacement during this period is-*

(A) 10 rad (B) 12.5 rad (C) 15 rad (D) 17.5 rad

Sol. Given $\omega_0 = 3$ rad/s, $\omega = 7$ rad/s

and $\alpha = 2$ rad/s^2

Applying $\omega^2 = \omega_0^2 + 2\alpha\theta$, we get

$(7)^2 = (3)^2 + 2(2)\theta$

$\therefore$ $\theta = \frac{49-9}{4} = 10$ rad.

COMPARATIVE TABLE

EQUATION OF LINEAR MOTION AND ROTATIONAL MOTION

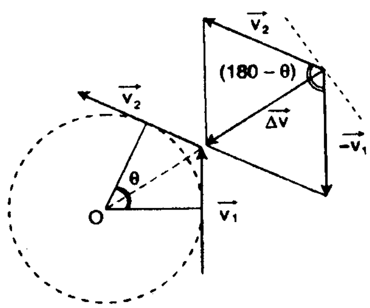
S.N.	LINEAR MOTION	ROTATIONAL MOTION
(i) With constant velocity	$a = 0$, $s = ut$	$\alpha = 0$, $\theta = \omega t$
(ii) With constant acceleration	(i) Average velocity	(i) Average angular velocity
	$v_{av} = \frac{v+u}{2}$	$\omega_{av} = \frac{\omega_1+\omega_2}{2}$
	(ii) Average acceleration	(ii) Average angular acceleration
	$a_{av} = \frac{v+u}{t}$	$\alpha_{av} = \frac{\omega_2+\omega_1}{t}$
	(iii) $s = v_{av}\, t = \frac{v+u}{2}t$	(iii) $\theta = \omega_{av} . t = \frac{\omega_1+\omega_2}{2}t$
	(iv) $v = u + at$	(iv) $\omega_2 = \omega_1 + \alpha t$
	(v) $s = ut + \frac{1}{2}at^2$	(v) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$
	(vi) $s = vt - \frac{1}{2}at^2$	(vi) $\theta = \omega_2 t - \frac{1}{2}\alpha t^2$
	(vii) $v^2 = u^2 + 2as$	(vii) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$
	displacement in nth sec.	Angular displacement in nth sec
	(viii) $S_n = u + \frac{1}{2}(2n - 1)a$	(viii) $\theta_n = \omega_1 + \frac{1}{2}(2n - 1)\alpha$

(iii) With variable acceleration

(i) $v = \frac{ds}{dt}$	(i) $\omega = d\theta/dt$
(ii) $\int ds = \int v dt$	(ii) $\int d\theta = \int \omega dt$
(iii) $a = \frac{dv}{dt} = v\frac{dv}{ds}$	(iii) $\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$
(iv) $\int dv = \int a dt$	(iv) $\int d\omega = \int \alpha\ dt$
(v) $\int v dv = \int a ds$	(v) $\int \omega d\omega = \int \alpha d\theta$

CENTRIPETAL ACCELERATION

It is predicted from the given fig. that direction of acceleration is towards centre. This "centre seeking" acceleration is called centripetal acceleration.



If direction of $\vec{\alpha}$ were anywhere but not towards centre, then it would definitely have one of it's components along tangent which would be responsible for change in speed. This change in speed will make motion non uniform.

MAGNITUDE OF CENTRIPETAL ACCELERATION :

PROOF FOR $a_r = (\upsilon^2/r)$

If a particle moving with uniform speed υ on a circle of radius r suffers angular displacement θ in time Δt, then change in its velocity

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1)$$

i.e. $$\Delta\upsilon = \sqrt{\upsilon^2 + \upsilon^2 + 2\upsilon^2\cos(180-\theta)}$$

or $$\Delta\upsilon = \upsilon\sqrt{2(1-\cos\theta)} = 2\upsilon\ \sin\ (\theta/2) \qquad ......(1)$$

It will be directed along the radius towards the centre. (Hence, also called radial acc.)

$$\theta = \frac{s}{r} = \frac{\upsilon\Delta t}{r}, \qquad \text{i.e.,} \quad \Delta t = \frac{r\theta}{\upsilon} \qquad ......(2)$$

So, in the light of Eqns. (1) and (2), radial acceleration

$$a_r = \frac{\Delta \upsilon}{\Delta t} = \frac{2\upsilon \sin(\theta/2)}{(r\theta/\upsilon)} = \frac{\upsilon^2}{r}\left[\frac{\sin(\theta/2)}{(\theta/2)}\right]$$

or $$a_r = \frac{\upsilon^2}{r} \qquad \left[\text{as} \lim_{(\theta/2)\to 0}\frac{\sin(\theta/2)}{(\theta/2)} \to 1\right]$$

$$a_c = \frac{\upsilon^2}{r} = \omega^2 r = \omega \times \omega r = \omega \times v$$

In vector form $\vec{a}_c = \vec{\omega} \times \vec{v}$

e.g. Direction of a_c (say-x) is perpendicular to both direction of $\vec{\omega}$ (+z) and $\vec{v}$ (+y).

Direction of $\vec{a}_c$ would be same as that of $\Delta\vec{v}$.

Ex.1 *A particle moves in a circle of radius 25 cm at two revolutions per second. The acceleration of the particle in m/s^2 is :*

(A) π^2 *(B) $2\pi^2$* *(C) $4\pi^2$* *(D) $8\pi^2$*

Sol. (C)

$r = 25 \times 10^{-2}$ m, f = frequency = 2/sec

$\therefore \quad \omega = 2\pi f = 4\pi$ rad/sec

Acceleration = $\omega^2 r = (4\pi)^2 \times 25 \times 10^{-2}$

$= 16 \times 25 \times 10^{-2}\, \pi^2\ m/s^2 = 4\pi^2\ m/s^2$.

Ex.2 *Certain neutron stars (extremely dense stars) are believed to be rotating at about 1 rev/sec. If such a star has a radius of 20 km, the acceleration of an object on the equator of the star will be (π^2 = 10)-*

(A) $20 \times 10^3\ m/s^2$ *(B) $120 \times 10^3\ m/s^2$* *(C) $8 \times 10^5\ m/s^2$* *(D) $4 \times 10^8\ m/s^2$*

Sol. (C)

Acceleration = $\omega^2 r = (2\pi f)^2 r = 4\pi^2 f^2 r$

$= 4\pi^2 \times 1 \times (2 \times 10^4) = 8 \times 10^5\ m/s^2$.

Ex.3 *The speed of revolution of a particle going around a circle is doubled and it's angular speed is halved. What happens to the centripetal acceleration ?*

(A) Remains unchanged *(B) Halved*

(C) Doubled *(D) Becomes four times.*

Sol. (A)

This is possible when radius of the circular path is made four times the initial value.

$$a = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2 = (r\omega)\omega = v\omega$$

Hence, when v is doubled & ω is halved, a remains unchanged.

CENTRIPETAL FORCE

(Though in kinematics, we don't study about force but this is a different kind of force which needs a little explanation here too).

As acceleration $\vec{a} = \dfrac{\vec{F}}{m}$, so force is a must for any kind of acceleration.

Force responsible for uniform circular motion of a particle is called centripetal force $\vec{F}_c$.

As $a_c = \dfrac{v^2}{r}$ and $a_c = \dfrac{F_c}{m}$ $\quad \therefore F_c = \dfrac{mv^2}{r} = m\omega^2 r$

In vector form, $\vec{F}_c = m\,\vec{a}_c = m\,(\vec{\omega} \times \vec{v}) = mv\omega \sin\theta\,\hat{n}$ ($\theta = 90^\circ$ for circular motion)

Centripetal force is not a real force. It is only the requirement. For circular motion, this requirement can be fulfilled by any kind of real forces like gravitational force, electrostatic force, magnetic force, tension of string, friction etc.

Ex. *Two particles of equal masses are revolving in circular paths of radii r_1 and r_2 respectively with the same time period. The ratio of their centripetal force is :*

(A) $\dfrac{r_1}{r_2}$ (B) $\sqrt{\dfrac{r_2}{r_1}}$ (C) $\left(\dfrac{r_1}{r_2}\right)^2$ (D) $\left(\dfrac{r_2}{r_1}\right)^2$

Sol. (A)

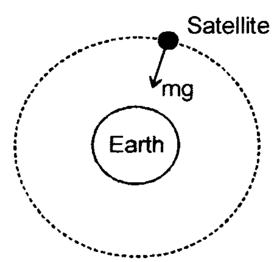
As $T_1 = T_2$

Hence, $\dfrac{2\pi r_1}{v_1} = \dfrac{2\pi r_2}{v_2}$ or $\dfrac{v_1}{v_2} = \dfrac{r_1}{r_2}$

$$\frac{F_1}{F_2} = \frac{mv_1^2}{r_1} \times \frac{r_2}{mv_2^2} = \left(\frac{v_1}{v_2}\right)^2 \times \frac{r_2}{r_1} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{r_2}{r_1} = \frac{r_1}{r_2}$$

ILLUSTRATIVE CASES (of real forces providing centripetal force)

(I) GRAVITATIONAL FORCE $\dfrac{mv^2}{r} = mg$



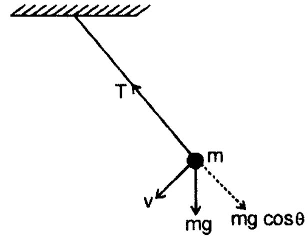
or gravitational attraction : $\dfrac{mv^2}{r} = \dfrac{GmM}{r^2}$

(where G is the gravitational const.)

(II) TENSION IN A STRING : $\frac{mv^2}{r} + mg\cos\theta = T$

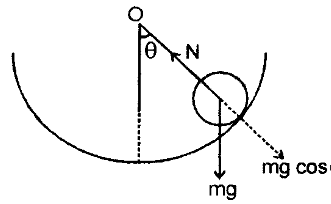
(where T is the tension)

(mg cos θ is the component of force due to gravity)



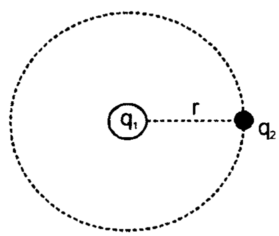
(III) NORMAL REACTION FORCE : $\frac{mv^2}{r} + mg\cos\theta = N$

(where N is the normal reaction)



(IV) ELECTROSTATIC FORCE : $\frac{mv^2}{r} = \frac{kq_1q_2}{r^2}$

(where q_1 and q_2 are the charges and r is the distance between them)



(a) If any how, during circular motion, suddenly v becomes zero but force responsible for providing centripetal acceleration is still present, then body will fall along radius towards centre.

(b) If any how, during circular motion, suddenly force responsible for providing centripetal acceleration vanishes (becomes 0) but v is still present, then body will be thrown out of the circle tangentially. eg. If a stone is tied to a string and rotated, then if we suddenly leave string then stone will be thrown out tangentially.

CENTRIFUGAL FORCE

When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer A, who is not sharing the motion along the circular path, the body appears to fly off tangentially at the point of release. To another observer B, who is sharing the motion along the circular path (i.e. the observer B also rotating with the body is released, it appears to B, as if it has been thrown off along the radius away from the centre by some force. This inertial force is called centrifugal force.) Its magnitude is equal to that of the centripetal force = $\frac{mv^2}{r}$. Centrifugal force is a fictitious force, which has to be applied as a concept only in a rotating frame of reference to apply Newton's Laws in that frame.

MISCELLANEOUS EXAMPLES BASED ON UNIFORM CIRCULAR MOTION

Ex.1 If a body is moving in a circle of radius r with a constant speed v, its angular velocity is-

(A) v^2/r (B) vr (C) v/r (D) r/v

Sol. (C)

$v = r\omega$

$\Rightarrow \quad \omega = \frac{v}{r}$ = constant [As v and r are constant].

Ex.2 Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same duration of time t. The ratio of the angular speed of the first to the second car is-

(A) $m_1 : m_2$ (B) $r_1 : r_2$ (C) 1 : 1 (D) $m_1r_1 : m_2r_2$

Sol. (C)

As time periods are equal, therefore ratio of angular speeds will be same.

$\omega = \frac{2\pi}{T}$.

Ex.3 A cyclist turns around a curve at 15 miles/hour. If he turns at double the speed, the tendency to overturn is-

(A) Doubled (B) Quadrupled (C) Halved (D) Unchanged

Sol. (B)

$F = \frac{mv^2}{r}$

$\Rightarrow \quad F \propto v^2$.

If v becomes double then F (tendency to overturn) will become four times.

Ex.4 A body of mass m is moving in a circle of radius r with a constant speed v. The force on the body is $\frac{mv^2}{r}$ and is directed towards the centre. What is the work done by this force in moving the body over half the circumference of the circle-

(A) $\frac{mv^2}{r} \times \pi r$ (B) Zero (C) $\frac{mv^2}{r^2}$ (D) $\frac{\pi r^2}{mv^2}$

Sol. (B)

Work done by centripetal force is always zero.

Ex.5 If a particle moves in a circle describing equal angles in equal times, its velocity vector-

(A) Remains constant (B) Changes in magnitude

(C) Changes in direction (D) Changes both in magnitude and direction

Sol. (C)

It is always directed along the tangent to circle.

Ex.6 A stone of mass m is tied to a string of length ℓ and rotated in a circle with a constant speed v. If the string is released, the stone flies-

(A) Radially outward (B) Radially inward

(C) Tangentially outward (D) With an acceleration $\frac{mv^2}{\ell}$

Sol. (C)

Stone flies in the direction of instantaneous velocity due to inertia.

Ex.7 A body is moving in a circular path with a constant speed. It has-

(A) A constant velocity (B) A constant acceleration

(C) An acceleration of constant magnitude (D) An acceleration which varies with time

Sol. (C)

Centripetal acceleration = $\frac{v^2}{r}$ = constant.

Direction keeps changing.

Ex.8 A motor cyclist going round in a circular track at constant speed has-

(A) Constant linear velocity (B) Constant acceleration

(C) Constant angular velocity (D) Constant force

Sol. (C)

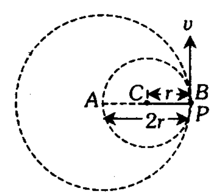
Linear velocity, acceleration and force vary in direction.

Ex.9 A particle P is moving in a circle of radius 'r' with a uniform speed v. C is the centre of the circle and AB is a diameter. When passing through B the angular velocity of P about A and C are in the ratio-

(A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1

Sol. (B)

Angular velocity of particle P about point A,



$$\omega_A = \frac{v}{r_{AB}} = \frac{v}{2r}$$

Angular velocity of P about point C,

$$\omega_C = \frac{v}{r_{BC}} = \frac{v}{r}$$

So, ratio $\frac{\omega_A}{\omega_C} = \frac{v/2r}{v/r} = \frac{1}{2}$.

PRACTICE QUESTIONS BASED ON UNIFORM CIRCULAR MOTION

Q.1 A body of mass m moves in a circular path with uniform angular velocity. The motion of the body has constant-
(A) Acceleration (B) Velocity (C) Momentum (D) Kinetic energy

Q.2 A cyclist taking turn bends inwards while a car passenger taking same turn is thrown outwards. The reason is-
(A) Car is heavier than cycle
(B) Car has four wheels while cycle has only two
(C) Difference in the speed of the two
(D) Cyclist has to counteract the centrifugal force while in the case of car only the passenger is thrown by this force.

Q.3 A car sometimes overturns while taking a turn. When it overturns, it is-
(A) The inner wheel which leaves the ground first
(B) The outer wheel which leaves the ground first
(C) Both the wheels leave the ground simultaneously
(D) Either wheel leaves the ground first

Q.4 Two bodies of mass 10 kg and 5 kg move in concentric orbits of radii R and r such that their periods are same. Then the ratio between their centripetal accelerations is-
(A) R/r (B) r/R (C) R^2/r^2 (D) r^2/R^2

Q.5 A car travels north with a uniform velocity. It goes over a piece of mud which sticks to the tyre. The particles of the mud, as it leaves the ground are thrown-
(A) Vertically upwards (B) Vertically inwards (C) Towards north (D) Towards south

Q.6 A train is moving towards north. At one place it turns towards north-east, here we observe that-
(A) The radius of curvature of outer rail will be greater than that of the inner rail.
(B) The radius of the inner rail will be greater than that of the outer rail.
(C) The radius of curvature of one of the rails will be greater.
(D) The radius of curvature of the outer and inner rails will be the same.

Q.7 The angular speed of a fly wheel making 120 revolutions/minute is-
(A) 2π rad/s (B) $4\pi^2$ rad/s (C) π rad/s (D) 4π rad/s

Q.8 A particle is moving on a circular path with constant speed, then its acceleration will be-
(A) Zero (B) External radial acceleration
(C) Internal radial acceleration (D) Constant acceleration.

Q.9 Cream gets separated out of milk when it is churned, it is due to-
(A) Gravitational force (B) Centripetal force
(C) Centrifugal force (D) Frictional force

Q.10 A particle moves in a circular orbit under the action of a central attractive force inversely proportional to the distance 'r'. The speed of the particle is-

(A) Proportional to r^2 (B) Independent of r (C) Proportional to r (D) Proportional to 1/r

Q.11 A 500 kg car takes a round turn of radius 50m with a velocity of 36 km/hr. The centripetal force is-

(A) 250 N (B) 750 N (C) 1000 N (D) 1200 N

Q.12 The angular velocity of a particle rotating in a circular orbit 100 times per minute is-

(A) 1.66 rad/s (B) 10.47 rad/s (C) 10.47 deg/s (D) 60 deg/s

ANSWERS

1.	(D)	**2.**	(D)	**3.**	(A)	**4.**	(A)	**5.**	(D)	**6.**	(A)
7.	(D)	**8.**	(C)	**9.**	(C)	**10.**	(B)	**11.**	(C)	**12.**	(B)