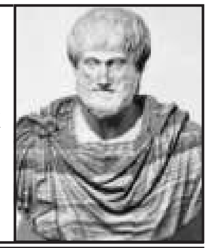
1 KINEMATICS

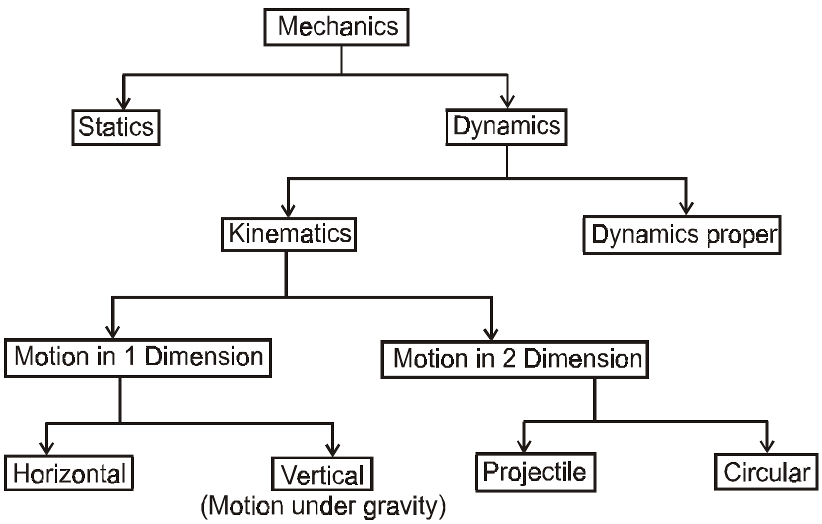
THIRD UMPIRE

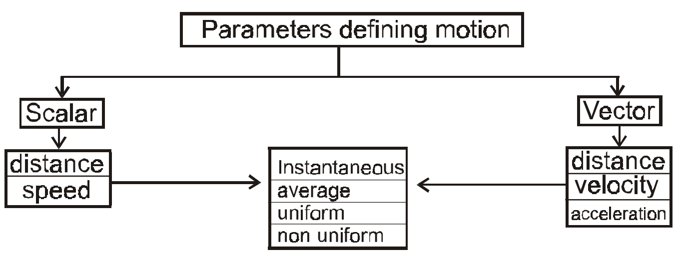
ARISTOTLE - The great Greek thinker - (384 B.C-322 B.C.)

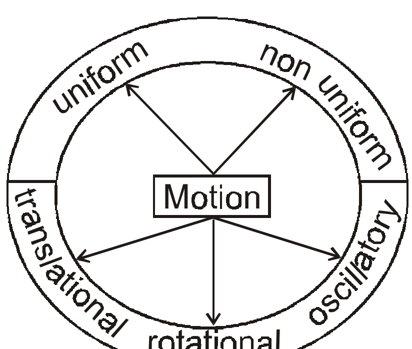
Aristotle was the first man to work on the field of *mechanics*. He held the view that if a body is moving, something external must be required to keep it in that state and prevent it from coming to a stop. The view was part of an elaborate framework of ideas developed by Aristotle on the motion of bodies in the universe. Most of the Aristotelian ideas on motion are now known to be wrong and need not concern us.



FIELD POSITION







BATTING LINE UP

(1) Rest & Motion

(2) Types of Motion

(3) Motion Defining Parameters

(a) Distance

(b) Displacement

(c) Speed

(d) velocity

(e) Acceleration

(4) Graphs

(5) Motion in 1-D (eqn. of motion)

(a) horizontal

(b) motion under gravity

(c) Relative 1-D

(8) Motion in 2-D

(a) Circular

(b) Projectile.

(c) Relative 2-D

TARGET JEE SYLLABUS

* **Kinematics (cartesian coordinate)**
* **Projectiles**
* **uniform circular motion**
* **Relative velocity**

MOTION DEFINING PARAMETERS

INTRODUCTION

Mechanics is the first branch of physics , so many scientists studied it in the initial days. It is classified into two parts :

(A) STATICS - It is the study of objects at rest.

(B) DYNAMICS - It is the study of objects in motion.

It can be further classified in two parts :

(I) KINEMATICS - Study of motion without taking into consideration the cause of motion.

(II) DYNAMICS PROPER - It is the study of motion by taking into account, the factors responsible for that motion.

MAIN CONTRIBUTORS TO MECHANICS

(A) ARISTOTLE **:** Born in 384 B.C., he was the first man to work in the field of mechanics. He held the view that if a body is moving, something external must be required to keep it in that state and prevent it from coming to a stop.

(B) GALILEO : He was born in Pisa, Italy in 1564 A.D. He invented the concept of acceleration. From experiments on motion of bodies on inclined planes or falling freely, he contradicted the Aristotelian notion that a force was required to keep a body in motion and that heavier bodies fall down faster under gravity.

(C) NEWTON : He was born in 1642 (the same year that Galileo died). He formulated the well-known laws of motion. He worked on theories of light and colour. He designed an astronomical telescope to carry out astronomical observations.

OPENING QUESTION

Q. *How can you predict the position of your friend just after he left your house ?*

Sol. Depends on path & vehicle and his driving pattern. Path and vehicle will help in determining types of motion and driving pattern will help in understanding motion defining parameters.

REST AND MOTION

PARTICLE

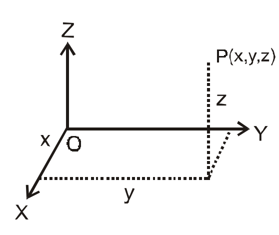
(A) DEFINITION OF PARTICLE

(i) A body of finite size of splitted parts may be considered as a particle only if all parts of the body undergo same displacement and have same velocity and acceleration.

(ii) When every part of an object undergoes same displacement and has same velocity and acceleration, we can describe its motion by the motion of any point of it.

(B) DEFINITION OF FRAME OF REFERENCE

To locate the position of a particle we need a frame of reference. A convenient way to do it, is to take three mutually perpendicular lines intersecting at a point called origin. The three lines are x-axis, y-axis, z-axis i.e. (x,y,z) are taken as the position co-ordinates of the particle.



(C) DEFINITION OF REST

If the position of an object does not change in space with respect to time (relative to an observer), it is said to be at rest.

(D) DEFINITION OF MOTION

If the position of an object in space changes with time (relative to an observer), it is said to be in motion. i.e. If all the three co-ordinates x,y and z of the particle remain unchanged as time passes, the particle is said to be at rest w.r.t. the frame, otherwise it will be in motion. Motion, therefore is a relative term i.e. it depends on *frame of reference* of observer.

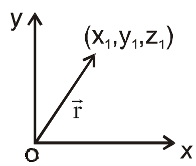
TYPES OF MOTION

Based on direction	Based on path followed by different particles of a body	Based on motion parameters
(1) One Dimensional (1D) or linear motion Position of the particle at any instant can be defined by only one parameter, say x. All possible positions lie on a ray only. E.g. : Motion of an aeroplane between two consecutive stations.	**(1)Translational motion** : If different parts of a body move identically so that the line joining any two points of it always remains parallel to itself, then the motion of body is said to be translatory motion. **Types of translational motion** (a) **Rectilinear** : When a body is moving along a straight line, the motion is rectilinear. (b) **Curvilinear** : When the body is moving along a curved path, the motion is said to be curvilinear.	**Motion with uniform velocity** * Velocity does not change with time. * it's straight line motion. * It's uniform motion. * Solving method: Use **DISTANCE = SPEED X TIME**
(2) Two Dimensional (2D) or Planer motion: 2 parameters (say x, y) are required to define the position of particle at any instant. All possible positions lie on a plane. Eg. : Motion of a bus between two stations. (assuming earth as flat), parabolic path followed by a projectile, circular path.	**(2) Rotational motion** Types of rotational motion **(a) Circular motion** If a particle, treated as a point, is moving on a circle, the motion is called *circular motion*. e.g. : planetary motion (actually elliptical). **(b) Rotatory motion** If the body cannot be treated as a point and all it's particles move simultaneously along circles (whose centres lie on a line called axis of rotation) by shifting through equal angle in a given time then motion is called *Rotatory motion*. eg. : Fan blades. **(c) Rolling motion** If the axis of rotation also moves, the motion is a combination of translatory and rotatory motion. This motion is called Rolling motion. eg. : Wheel of a moving bus.	**Motion with uniform acceleration** * change of velocity is uniform * It's non uniform motion. * Solving method : Use equations of motion : $v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$
(3)Three Dimensional (3D) : 3 parameters (say x, y, z) are required to define the position. Position confined in the volume. Eg. Helical (DNA Helix) (defined by pitch and radius), flying mosquito (random).	**(3) Oscillatory or Vibratory motion** : If the periodic motion is constrained within certain limits i.e. to and fro or up and down, then it is called oscillatory motion. If amplitude is microscopic then it's usually called **vibratory motion**. e.g. pendulum of a wall clock.	**Motion with non-uniform acceleration :** * Change of velocity is non-uniform * It's non-uniform motion. * Solving method : Use differentiation and integration.

MOTION DEFINING PARAMETERS

BASIC MOTION DEFINING PARAMETERS

POSITION OF AN OBJECT



POSITION VECTOR

It is a vector from origin to the object which represents the position of object with respect to origin.

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $|r| = \sqrt{x^2 + y^2 + z^2}$

DISTANCE AND DISPLACEMENT

DEFINITIONS

(A) DISTANCE (Denoted by x or s)

It is the scalar quantity giving *actual length of the path* (irrespective of direction) of motion for moving object.

- Dimension $[M^0 L^1 T^0]$, units : SI/MKS $\Rightarrow$ meter(m), cgs $\rightarrow$ centimeter (cm).

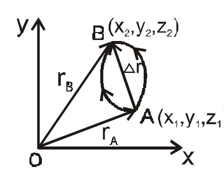
(B) DISPLACEMENT

It is the vector quantity whose magnitude is the *shortest distance between initial and final position of the object* (whatever be the path) and direction is specified by the ray from initial position to final position.

1. It is the change in position vector (Δr) or vector joining initial and final position (r_{AB})

   i.e. $\Delta\vec{r} = \vec{r}_f - \vec{r}_i = \vec{r}_B - \vec{r}_A = \vec{r}_{BA}$

   $= (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$



2. There can be two types of vector notations :

   (i) $\Delta\vec{r} = \vec{r}_{AB}$ (where AB means vector from A $\rightarrow$ B)

   (ii) $\vec{r}_{BA}$ (here also direction of vector is from A to B but writing B before A means we are giving position of B wrt A, as only r_B will mean position of A wrt origin, we will stick to this)

3. Dimension $[M^0 L^1 T^0]$, unit : MKS - meter (m), cgs - centimeter (cm).
4. Displacement of an object remains unchanged by shifting the origin of the position vector.

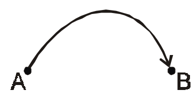
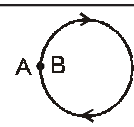
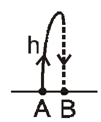
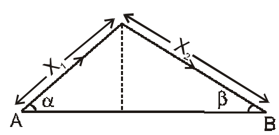
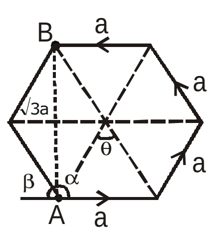
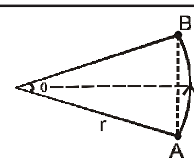
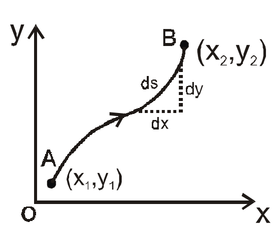
DIFFERENCE BETWEEN DISTANCE AND DISPLACEMENT

Distance	**Displacement**
(1) It is scalar quantity.	(1) It is vector quantity
(2) It can never be negative.	(2) It can be positive or negative.
(3) For a moving object, it always increases with time.	(3) The magnitude of displacement can decrease with time if object is moving towards initial position.
(4) It is path dependent.	(4) It is path independent.
(5) For two points A and B, it can have many values depending on the path chosen.	(5) For two fixed points A and B, it is single valued.
Ex. *Bus route between two stations gives distance between them.*	***Ex.*** *Direct Aeroplane route between two stations gives displacement vector between them.*

MOTION DEFINING PARAMETERS

ILLUSTRATIONS

(INITIAL POSITION A, FINAL POSITION B, PATH SHOWN BY ARROW)

(i) A straight path		Distance = | displacement | = x
(ii) Half circle (radius r)		Distance = πr, | displacement | = 2r
(iii) Full circle (radius r)		Distance = $2\pi r$, displacement = 0
(iv) Stone thrown up to height h, back to ground.		Distance = 2h, displacement = 0
(v) Inclined up and down.		Distance = $x_1 + x_2$ |displacement| = $x_1 \cos\alpha + x_2 \cos\beta$
(vi) Regular polygon of n sides, out of which m sides travelled. eg. Hexagon		Distance = 4a | displacement | = $\sqrt{3}$ a (Use formula $\theta = \beta = \frac{2\pi}{n}$, $\alpha = \pi - \beta = \frac{(n-2)\pi}{n}$)
(vii) Circular Arc		distance = arc = θr | displacement | = 2r sin (θ/2)
(viii) Curve y = f(x) $(ds)^2 = (dx)^2 + (dy)^2$ $ds = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} dx$		Distance, S = $\int_{x_1}^{x_2} \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx$ Displacement= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

MOTION DEFINING PARAMETERS

<u>CONCEPT</u> | Displacement | $\leq$ Distance

<u>MISCONCEPT</u> Modulus of displacement vector gives distance.

<u>CLARIFICATION</u> Displacement equals minimum possible distance and not any distance of any path.

GRAPHICAL INTERPRETATION

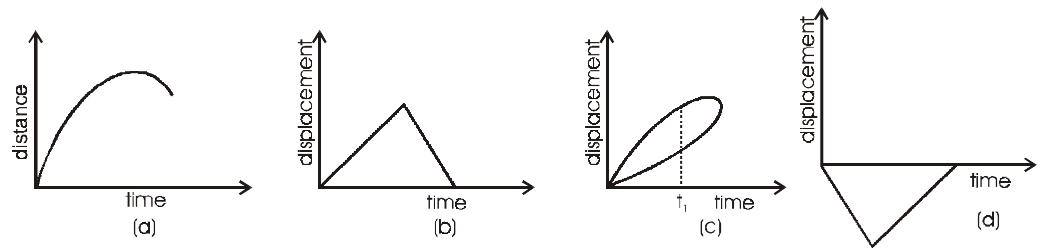
(a) Displacement - time graph does not give trajectory

(trajectory = actual path followed by particle which is given by x - y graph)

(b) Whenever we talk of displacement-time graph, it means |displacement |-time graph because vector portion cannot be so easily represented in simple graph. Though, positive and negative signs are used with this magnitude to represent two opposite directions.

<u>ILLUSTRATION</u>

Q.1 *which of the following graph(s) is/are not possible ?*



(A) only a,c *(B) only a,b,c* *(C) only b,c,d* *(D) only c*

Sol. (B) a,b,c are not possible, because

* graph (a) : distance - time graph is always increasing (for moving body)

* graph (b) : displacement time graph cannot have sharp curve ideally.

* graph (c) : two displacements are not possible at a single instant.

EXAMPLES BASED ON DISTANCE - DISPLACEMENT

Q.1 A lost ship in sea firstly moved 2 km in North West, then $\sqrt{2}$ km southward and then 4 km in North East direction. Find the displacement of the ship.

(A) $\sqrt{10}$ km, $\tan^{-1}\frac{1}{2}$ (B) $2\sqrt{5}$ km, $\tan^{-1}\frac{1}{2}$ (C) $\sqrt{10}$ km, $\tan^{-1}2$ (D) $2\sqrt{5}$ km, $\tan^{-1}2$

Sol. (C)

$\Delta r_1 = -2\cos 45\,\hat{i} + 2\sin 45\,\hat{j}$

$\Delta r_2 = -\sqrt{2}\,\hat{j}$

$\Delta r_3 = 4\cos 45\,\hat{i} + 4\sin 45\,\hat{j}$

$\Delta\vec{r} = \Delta r_1 + \Delta r_2 + \Delta r_3 = \sqrt{2}\,\hat{i} + 2\sqrt{2}\,\hat{j}$

$\therefore\ |\Delta r| = \sqrt{10}$ km, $\tan\theta = \frac{2\sqrt{2}}{\sqrt{2}} = 2$

MOTION DEFINING PARAMETERS

Q.3 A particle suffers three displacements by 4m in the northward, 2m in the south-east and 2m in the south-west directions. What is the displacement of the particle and what is the distance covered by it ?

Sol. Taking a frame of reference with the x-axis in the eastward and the y-axis in the northward direction

$\vec{s}_1 = 4\hat{j}$; $\vec{s}_2 = 2 \cos 45^\circ \hat{i} - 2 \sin 45^\circ \hat{j}$

$\vec{s}_3 = -2 \cos 45^\circ \hat{i} - 2 \sin 45^\circ \hat{j}$

$\vec{s} = \vec{s}_1 + \vec{s}_2 + \vec{s}_3 = 4\hat{j} + \sqrt{2}\hat{i} - \sqrt{2}\hat{j} - \sqrt{2}\hat{i} - \sqrt{2}\hat{j}$

$\therefore \vec{s} = (4 - 2\sqrt{2})\hat{j} = 4 - 2\sqrt{2} = 1.17$ m (northward)

SPEED

DEFINITION

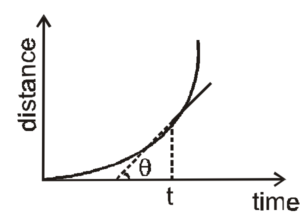
1. It is the distance covered by particle in one sec.
2. It's a scalar quantity.
3. It can never be negative
4. Dimension = $[M^0 L^1 T^{-1}]$, Unit : (MKS) $\Rightarrow$ m/s, km/hr (CGS) $\Rightarrow$ cm/sec.

Unit conversion formula $\Rightarrow$ x km/hr = $\left(\frac{5}{18}\right)$ x m/sec.

TYPES OF SPEED

(A) <u>INSTANTANEOUS SPEED (V_{INS})</u>

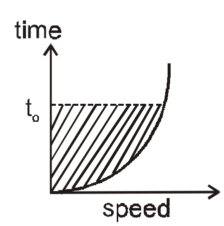
It's the speed of a particle at a particular instant of time or position.



$$V_{ins} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$\therefore$ Slope of the tangent at any point in distance time graph gives speed at that point $\therefore v_t = \frac{dx}{dt} = \tan\theta$.

From the above formula, it can also be predicted that



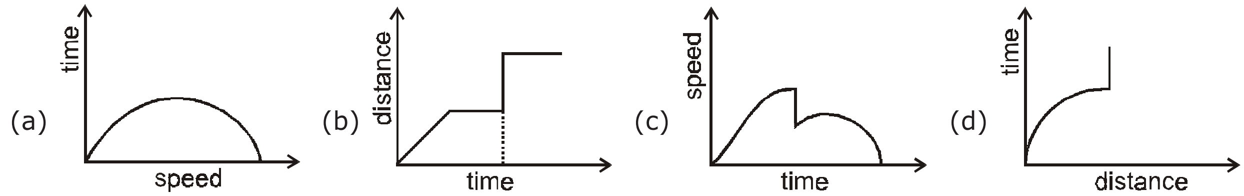
Total distance covered, x = $\int dx = \int v\,dt$.

As vdt represents area

$\therefore$ Area under speed time graph over time axis gives distance covered.

EXAMPLE :

Q. *Which of the following graph(s) is/are ideally not possible.*



Sol. Ideally, fast changes in speed at an instant and infinite speed are not possible, so graphs (a), (b), (c) are not possible ideally. Though casually, this can be used most of the time theoretically.

<< method of finding inst. speed if distance is given as a function of time >>

Differentiating the x(t) function wrt t will give you v(t) function. Putting value of t in this function will give you instantaneous velocity at that moment.

EXAMPLE :

Q. *The displacement of a car is given by the relation, $x = 6t^2 + 5t + 4$, where t is time. Find the velocity of car at t = 5 sec.*

Sol. Given, $x = 6t^2 + 5t + 4$

then, $v = \frac{dx}{dt} = 12t + 5$ $\therefore$ velocity (at t = 5 sec) = 60 + 5 = 65 m/s.

<< method of finding distance travelled if speed is given as a function of time >>

integrating the v(t) function wrt t will give you distance as a function of time i.e. x(t). Otherwise if v(t) can be plotted on graph then area under the curve over time axis will give you distance.

Total distance covered, $x = \int dx = \int v\,dt$.

EXAMPLE :

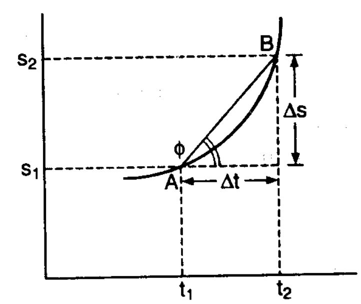
Q. *Speed of a particle is given as $v = 4 + 3t^2$. Find distance covered in t = 0 to t = 3 sec.*

Sol. $x = \int v\,dt = \int (4+3t^2)\,dt = \left[4t + t^3\right]_0^3 = 39$ unit.

(B) <u>AVERAGE SPEED</u> (v_{av}) or (< v >)

If a body covers total distance ΔS over a certain time period Δt, then

$$V_{av} = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{\Delta s}{\Delta t}$$



Slope of chord between two different times t_1 (point A) and t_2 (point B) in distance time graph gives average speed.

<< method of finding avg. speed when motion is broken into several parts >>

If a particle travels x_1, x_2, ... x_n at speeds v_1, v_2, ... v_n taking time t_1, t_2 .... t_n respectively then

$\Delta S = x_1 + x_2$ .... $+ x_n$ or $\Delta S = v_1 t_1 + v_2 t_2 + v_3 t_3$ .... $+ v_n t_n$. (any one will be known)

$\Delta t = t_1 + t_2 + \ldots\ldots\ldots\ldots\; t_n$ or $\Delta t = \frac{x_1}{v_1} + \frac{x_2}{v_2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{x_n}{v_n}$. (any one will be known)

then, substituting above Δs and Δt in equation, $v_{av} = \frac{\Delta s}{\Delta t}$. It will give average speed.

LET'S TAKE THE FOLLOWING POSSIBLE CASES

CASE -I values of distance and time interval for different parts of motion given individually. WHAT TO DO **:** Simply add all x, all t between two specified positions and find $V_{AV} = \frac{\Delta s}{\Delta t}$.

Q. *A bus goes from station A to D through B, C. The distance and time are given in fig.*

$A \xrightarrow[2t]{x} B \xrightarrow[t]{x} C \xrightarrow[t]{2x} D$. *Find the average speed between (a) AC (b) BD (c) AD.*

Ans. (a) $\frac{2x}{3t}$ (b) $\frac{3x}{2t}$ (c) $\frac{x}{t}$

CASE - II

Velocity is given alongwith either x or t. Find $\Delta s = \sum$ vt and $\Delta t = \sum \frac{x}{v}$

Q. *A body travels at a uniform speed of 3 ms^{-1} for 20 seconds, then it stops for 10 seconds. Then it travels at uniform speed of 2 ms^{-1} upto 80 m. It's average speed during the motion is-*
(A) 2 ms^{-1} (B) 2.5 ms^{-1} (C) 3 ms^{-1} (D) 3.5 ms^{-1}

Ans. (A)

CASE - III

Instantaneous speed and time are given. Find $\Delta s = \int v\,dt$ and then $V_{av} = \frac{\Delta s}{\Delta t}$.

Q. *If V(t) = 6t^2 + 2t + 1, then find the avg speed during period t = 0 to t = 3 sec.*
(A) 16 m/s (B) 8 m/s (C) 10 m/s (D) 22 m/s

Ans. (D)

CASE - IV

If $x_1 = x_2 = \ldots\ldots\ldots\ldots = x_n$ in above case then prove that Average speed is harmonic mean (HM) of individual speed and for n = 2, $v_{av} = \frac{2v_1v_2}{v_1 + v_2}$.

Q. *A man travels first half distance with speed 6 km/hr and second half distance with speed 2 km/hr. Find Average speed.*
(A) 4 km/hr. (B) 3 km/hr. (C) 1.5 km/hr. (D) 4.5 km/hr.

Ans. (B)

CASE - V

If $t_1 = t_2$ .... $= t_n$ in above case, then prove that $v_{av} = \frac{\Sigma v_i}{n}$ i.e. Average speed is arithmetic mean (AM) of individual speeds and for n = 2, $v_{av} = \frac{v_1 + v_2}{2}$.

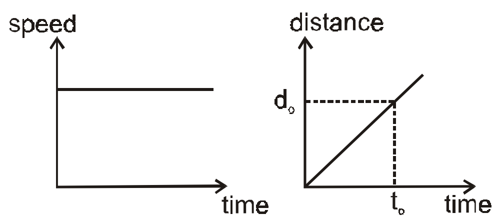
Q. *A man travels first half of his journey time with speed 6 km/hr and second half of his journey time with speed 2 km/hr. Find Average speed.*

(A) 4 km/hr. (B) 3 km/hr. (C) 1.5 km/hr. (D) 4.5 km/hr.

Ans. (A)

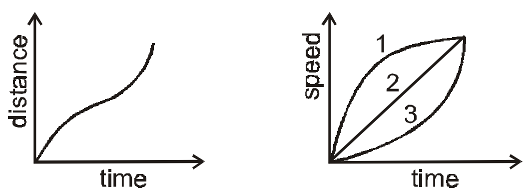
UNIFORM SPEED

1. Speed, which is not changing with time is called uniform speed.
2. Uniform speed is possible even in 1-D, 2-D or 3-D motion.
3. Slope at every point in x-t graph is same so it's a straight line in x-t graph.



NON-UNIFORM SPEED

1. When speed of particle changes with time, then motion is called non-uniform motion.
2. Curve 1, 2, 3 all represent non-uniform speed.



EXAMPLES BASED ON SPEED

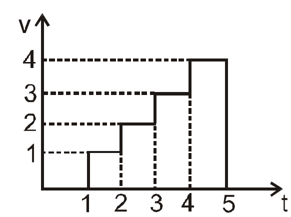
Ex.1 If t = $\sqrt{x}$ +3 then find the displacement of the particle when its velocity is zero.

Sol. $x = t^2 - 6t + 9 = (t - 3)^2$ $\therefore$ $v = \frac{dx}{dt} = 2t - 6$

v = 0 at t = 3 sec. so, at t = 3 sec. $x = 3^2 - 6\times3 + 9 = 0$

Ex.2 If v = [t] where [] is the greatest integer function and v is in m/s, find the distance travelled during time t = 2 sec to t = 5 sec.

(A) 6 m (B) 14 m (C) 54 m (D) 9 m



Ans. (D)

EXAMPLE BASED ON AVERAGE SPEED

Ex.1 A point travelling along a straight line traverses one third the distance with a velocity v_o. The remaining part of the distance was covered with velocity v_1 for half the time and with velocity v_2 for the other half of the time. The mean velocity of the point averaged over the whole time of motion will be-

(A) $\frac{v_o(v_1+v_2)}{3(v_1+v_2+v_o)}$ (B) $\frac{3v_o(v_1+v_2)}{v_1+v_2+v_o}$ (C) $\frac{v_o(v_1+v_2)}{v_1+v_2+4v_o}$ (D) $\frac{3v_o(v_1+v_2)}{v_1+v_2+4v_o}$

Sol. Let s be the total distance travelled. Let (s/3) distance be covered in time t_1 while the remaining distance (2s/3) in time t_2 second.

Now $\left(\frac{s}{3}\right) = v_o t_1$ or $t_1 = \frac{s}{3v_o}$

and $\left(\frac{2s}{3}\right) = v_1\left(\frac{t_2}{2}\right) + v_2\left(\frac{t_2}{2}\right)$ or $t_2 = \frac{4s}{3(v_1+v_2)}$

Average velocity $= \frac{s}{t_1+t_2} = \frac{s}{\frac{s}{3v_o}+\frac{4s}{3(v_1+v_2)}} = \frac{3v_o(v_1+v_2)}{v_1+v_2+4v_o}$

Hence, correct answer is (D).

VELOCITY

DEFINITION

1. Displacement in unit time is called velocity.
2. It is vector quantity.
3. Dimension $[M^0 L^1 T^{-1}]$, Unit : (MKS) $\Rightarrow$ m/s, (CGS) $\Rightarrow$ cm/s.

TYPES OF VELOCITY

(A) <u>INSTANTANEOUS VELOCITY</u>

1. It is velocity of a particle at a particular instant.
2. $V_{ins} = \lim_{\Delta t\to 0}\left(\frac{\Delta \vec{r}}{\Delta t}\right) = \frac{d\vec{r}}{dt}$

$\therefore$ Slope of the tangent at any point gives value of instantaneous velocity at that point. i.e. v = tan θ.

EXAMPLE :

Q. *The displacement of a particle is given by $\vec{r} = (t^2 + 2t + 1)$m (where t is the time period). Calculate the velocity of the particle as a function of time.*

Sol. $V_{ins} = \lim_{\Delta t\to 0}\left(\frac{\Delta \vec{r}}{\Delta t}\right) = \frac{d\vec{r}}{dt} = (2t + 2)$ m/s.

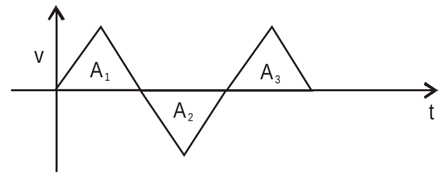
3. It can also be defined as rate of change of position vector.
4. For the curved path, instantaneous velocity is always tangential to the path followed.

5. Total displacement of particle in time t is $\Delta\vec{r} = \int_0^t \vec{v}\, dt$

$\therefore$ Area under velocity vs time graph and time axis with proper algebraic sign gives displacement, while without sign gives distance.

In graph, distance = $A_1 + A_2 + A_3$

$|\text{displacement}| = A_1 - A_2 + A_3$



(B) <u>AVERAGE VELOCITY</u> ($\vec{v}_{av}$ or < $\vec{v}$ >)

1. It is the ratio of displacement with time.

2. $\vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

3. Direction of any velocity is same as that of displacement vector.

<< method of finding avg. velocity when velocity is given as a function of time >>

$$\bar{v}_{av} = \frac{\int_0^t \vec{v}\,dt}{\int_0^t dt}$$

EXAMPLE :

Q. *The velocity of a moving car varies as a function of time and the relation is given by v = 2at + b, where a & b are constants. Calculate the average velocity of the car during time t = 0 to t = t.*

Sol. $\Delta\vec{r} = \int_o^t (2at + b)dt$ = $(at^2 + bt)$, so $\vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$ = $(at^2 + bt)/t = at + b$.

(C) <u>UNIFORM VELOCITY</u>

1. When velocity of a particle does not change w.r.t. time, then it's velocity is known as uniform velocity.
2. It is possible only when particle moves along a straight line without reversing it's path.

(D) <u>NON-UNIFORM VELOCITY.</u>

1. When velocity of particle changes w.r.t. time, then it is called non uniform velocity.
2. There can be three reasons for this non-uniformity.

(a) When only magnitude changes - eg. motion of a particle in straight line with constant acceleration.

(b) When only direction changes - eg. uniform circular motion.

(c) When both change - eg. projectile motion, vertical circular motion.

MOTION DEFINING PARAMETERS

CONCEPTS REGARDING SPEED AND VELOCITY

1. For uniform speed (velocity) motion, average speed (velocity) is equal to instantaneous speed (velocity) but converse may not be true.

   i.e. accidentally at one place if $v_{in} = v_{av}$, then it does not necessarily mean that it's a uniform motion.

2. If velocity is constant, then speed is also constant but converse may not be true.

Ex. *Circular motion, where speed is const but velocity keeps on changing due to change in direction.*

3. All the differences between distance and displacement are applicable between average speed and average velocity.

   i.e. $v_{av} \geq |\vec{v}_{av}|$, $v_{av} > 0$ but $\vec{v}_{av} > = < 0$, single valued, path independent.

4. Magnitude of instantaneous velocity is called instantaneous speed i.e. $v = |\vec{v}| = \left|\frac{d\vec{r}}{dt}\right|$ but speed $\neq \frac{d|\vec{r}|}{dt}$.

   because $d|\vec{r}|$ is change in magnitude of position vector (and not the magnitude of change in position vector which is $|d\vec{r}|$).

Ex. *In uniform circular motion $d|\vec{r}| = 0$ but speed $\neq 0$.*

5. If direction of velocity changes, then value of | displacement | will be different from value of distance covered.

ILLUSTRATION

Q. *Which of the following statements are true for a moving body ?*

(A) If its speed changes, its velocity must change and it must have some acceleration.

(B) If its velocity changes, its speed must change and it must have some acceleration.

(C) If its velocity changes, its speed may or may not change and it must have some acceleration.

(D) If its speed changes but direction of motion does not change, its velocity may remain constant.

Sol. (A), (C)

EXAMPLE BASED ON VELOCITY

Q. The velocity of a car is given by $\vec{v} = (4t + 2)$ m/s, where t is the time period. Find the total displacement of the car in time t.

Sol. $\Delta\vec{r} = \int_0^t \vec{v}\, dt = (2t^2 + 2t)$m.

EXAMPLE BASED ON AVERAGE VELOCITY

Q. A table has its minute hand 4.0 cm long. The average velocity of the tip of the minute hand between 6.00 a.m. to 6.30 a.m. and 6.00 a.m. to 6.30 p.m. will respectively be- (in cm/s)

(A) 4.4×10^{-3}, 1.8×10^{-4} (B) 1.8×10^{-4}, 4.4×10^{-3}

(C) 8×10^{-4}, 4.4×10^{-3} (D) 4.4×10^{-3}, 8×10^{-4}

Sol. (A) At 6.00 a.m. the tip of the minute hand is at 12 mark and at 6.30 a.m. or 6.30 p.m., it is 180° away. Thus, the straight line distance between the initial and final positions of the tip is equal to the diameter of the clock. So, displacement = 2 R = 2 x 4 cm = 8 cm.

Time taken from 6 a.m. to 6.30 a.m. is 30 minutes = 1800 s. The average velocity is v_{av} =

$$\frac{\text{Displacement}}{\text{time}} = \frac{8.0\,\text{cm}}{1800\text{s}} = 4.4 \times 10^{-3} \text{ cm/s}$$

Again time taken from 6 am to 6.30 p.m. = 12 hrs + 30 minutes = 45000 s

$$\therefore\ v_{av} = \frac{\text{Displacement}}{\text{time}} = \frac{8}{45000} = 1.8 \times 10^{-4} \text{ cm/s}$$

Hence, correct answer is (A).

ACCELERATION

DEFINITION

1. Rate of change of velocity is called acceleration.
2. It is a vector quantity, dimension $[M^0 L^1 T^{-2}]$, unit $\Rightarrow$ m/s^2.
3. $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$
4. If constant force F is acting on mass m, then $\vec{a} = \frac{\vec{F}}{m}$.
5. There is no definite relation between direction of velocity vector and direction of acceleration vector.
6. If only direction of velocity changes, then $\vec{a}$ is perpendicular to $\vec{v}$.
7. **3 ways of change in velocity are**

CASE I : Only magnitude of velocity changes.

Ex. *Retardation : direction of velocity is opposite to that of acceleration.*

CASE II : Only direction of velocity changes.

Ex. *Circular motion : direction of velocity is tangential but acceleration is towards center.*

CASE III : Both magnitude and direction of velocity changes

Ex. *Projectile motion : velocity is changing but acceleration is always downward.*

TYPES OF ACCELERATION

(A) INSTANTANEOUS ACCELERATION

1. Acceleration of a particle at a particular instant is known as instantaneous acceleration.
2. $\vec{a} = \lim_{\Delta t \to 0} \left| \frac{d\vec{v}}{dt} \right| = \frac{d\vec{v}}{dt} = \frac{d^2 s}{dt^2}$

EXAMPLE :

Q. *A Bus travels from New York to San Francisco with a velocity given by the relation :-*
$v = f(t) = (7t^2 + 6)$. Find the acceleration $\vec{a}(t)$ in terms of time.

Sol. Given, $v = 7t^2 + 6$

so, $\vec{a}(t) = \frac{d\vec{v}}{dt} = 14\,t.$

3. $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{v}}{ds} \cdot \frac{d\vec{s}}{dt} = \frac{\vec{v}d\vec{v}}{ds}$

(B) AVERAGE ACCELERATION

1. If change in velocity is $\Delta\vec{v}$ in time Δt, then $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$.
2. The direction of ang. acceleration vector is the direction of the change in velocity vector.

EXAMPLE :

Q. *A particle goes along a quadrant AB of a circle of radius 5 cm with a constant speed 2.5 cm/s as shown. Find the average velocity and average acceleration over the interval AB.*

Sol. Time taken = $\frac{\text{distance}}{\text{speed}} = \frac{3.14 \times 5}{2 \times 2.5}$ = 3.14 s.

Average velocity = $\frac{\text{displacement}}{\text{time}}$

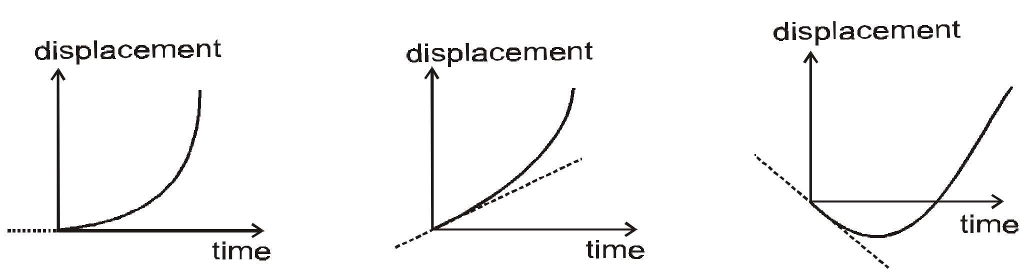
$= \frac{AB}{\text{time}} = \frac{\sqrt{5^2 + 5^2}}{3.14}$ m/s = 2.252 m/s

Average Acceleration = $\frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_B - \vec{v}_A}{\Delta t} = \frac{\sqrt{(2.5)^2 + (2.5)^2}}{3.14}$ m/s² = 1.126 m/s²

The average velocity is directed along AB and the average acceleration is directed perpendicular to AB towards O.

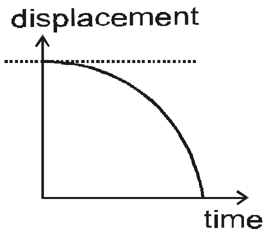
(C) UNIFORM ACCELERATION

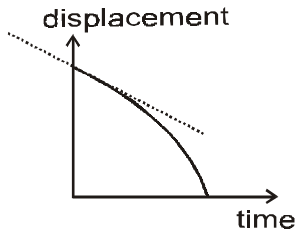
1. If the magnitude and direction of acceleration is not changing with time then acceleration is uniform. eg. projectile motion.
2. Uniform acceleration does not mean uniform motion because velocity is still changing.
3. Uniform acceleration does not necessarily imply that particle is moving in one direction only. eg. projectile motion.
4. Displacement - time graph for positively accelerated motion is upward opening parabola

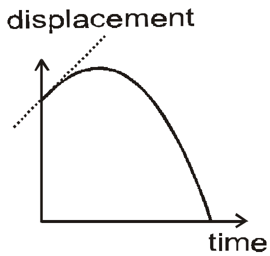


$x_0 = 0$, $v_0 = 0$, $a > 0$ $x_0 = 0$, $v_0 > 0$, $a > 0$ $x_0 = 0$, $v_0 < 0$, $a > 0$

while for negatively accelerated motion, it is downward opening parabola.







$x_0 \neq 0$, $v_0 = 0$, $a < 0$ $\qquad$ $x_0 \neq 0$, $v_0 < 0$, $a < 0$ $\qquad$ $x_0 \neq 0$, $v_0 > 0$, $a < 0$

NOTE : This topic will be dealt in detail in next chapter : motion in one dimension and motion under gravity

(D) NON-UNIFORM ACCELERATION

1. When acceleration (either magnitude or direction or both) changes with time then it's called non-uniform acceleration.

TABLE OF DIFFERENCES

	speed	velocity	acceleration
(1) Instant.	$v = \frac{dx}{dt} = \lvert \vec{v} \rvert$	$\vec{v} = \left[\frac{d\vec{r}}{dt}\right]_{\text{at time t}}$	$\vec{a} = \left[\frac{d\vec{v}}{dt}\right]_{\text{at time t}}$
(2) average	$v_{av} = \frac{\text{total distance}}{\text{total time}}$ $v_{av} \neq \lvert \vec{v}_{av} \rvert$	$\vec{v}_{av} = \frac{\text{total displacement}}{\text{total time}}$ $= \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t}$	$\vec{a} = \frac{\text{Net change in velocity}}{\text{total time}}$ $= \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$
(3) uniform	$\lvert v \rvert$ is not changing but $\vec{v}$ may change with time.	neither direction nor magnitude of velocity is changing so necessarily one - D motion.	acceleration is not changing but velocity (direction and/or magnitude) is changing. (v is function of first power of t).
(4) non-uniform	speed is increasing or decreasing with time so acceleration is there.	If any or both of direction and magnitude of velocity is changing then it is non-uniform. So acc. is there.	Rate of change of velocity is changing i.e. (v is function of t^n. where n>1)

MOTION DEFINING PARAMETERS

EXAMPLES BASED ON INSTANTANEOUS ACCELERATION

Q.1 A car is moving with a velocity $\vec{v} = 2x^2 + 3x$. (where x is the instantaneous displacement). Can you find the acceleration of the car in terms of displacement ?

Sol. Velocity of the car is given as

$v = 2x^2 + 3x$

Now differentiating v w.r.t. x, we get :

$\frac{d\vec{v}}{dx} = \frac{d}{dx}(2x^2 + 3x) = (4x + 3)$.

Acceleration = $\vec{v}\frac{d\vec{v}}{dx} = (2x^2 + 3x)(4x + 3)$

$\therefore\ \vec{a} = 8x^3 + 18x^2 + 9x$.

Area under a-t graph gives change in velocity.

Q.2 What is meant by $\frac{d|\vec{v}|}{dt}$ and $\left|\frac{d\vec{v}}{dt}\right|$? Can these be equal? Can -

(i) $\frac{d|\vec{v}|}{dt} = 0$, while $\left|\frac{d\vec{v}}{dt}\right| \neq 0$

(ii) $\frac{d|\vec{v}|}{dt} \neq 0$, while $\left|\frac{d\vec{v}}{dt}\right| = 0$

Sol. $\frac{d|\vec{v}|}{dt}$ means the time rate of change of speed and $\left|\frac{d\vec{v}}{dt}\right|$ means the magnitude of acceleration.

These two can be equal in two situations :

- When a particle moves with uniform velocity
- When a particle moves with constant acceleration along a straight line

(i) $\frac{d|\vec{v}|}{dt} = 0 \Rightarrow$ speed = constant

$\left|\frac{d\vec{v}}{dt}\right| \neq 0 \Rightarrow$ |acceleration| $\neq 0$

It happens only when a particle moves with constant speed along a circular path of constant radius.

(ii) $\left|\frac{d\vec{v}}{dt}\right| = 0 \Rightarrow$ |acceleration| = 0

$\frac{d|\vec{v}|}{dt} \neq 0 \Rightarrow$ speed $\neq$ constant

For zero acceleration, speed cannot change.

Therefore, this case is not possible.

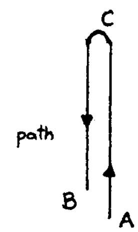
EXAMPLE BASED ON AVERAGE ACCELERATION

Q. A bird flies north at 20 m/s for 15s. It rests for 5s and then flies south at 25 m/s for 10s. For the whole trip, find-

(1) The average speed ;

(2) The average velocity ;

(3) The average acceleration.



Sol. Distance travelled towards north

$\Rightarrow$ AC = 20 m/s × 15s = 300 m.

distance travelled towards south

$\Rightarrow$ CB = 25 m/s × 10 s = 250 m.

$$\text{Average Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{300+250}{15+5+10}\ \text{m/s} = 18.34\ \text{m/s}.$$

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{300-250}{15+5+10} = 1.67\ \text{m/s}.$$

$$\text{Average Acceleration} = \bar{a}_{av} = \frac{\Delta\bar{v}}{\Delta t} = \frac{\vec{v}_f - \bar{v}_i}{\Delta t}$$

$$= \frac{(-25)-(+20)}{30}\ \text{m/s}^2 = -1.5\ \text{m/s}^2.$$

EXAMPLES BASED ON NON-UNIFORM ACCELERATION

Q.1 A ferry boat moves with constant velocity u_0 =8 m/s for 60 s. It then shuts off its engines and coasts. Its coasting velocity is given by $u = u_0 t_1^2/t^2$, where t_1 = 60 s. What is the displacement of the boat from t = 0 to $t \to \infty$?

Sol. **Picture the Problem** This velocity function is shown in Figure. The total displacement is calculated as the sum of the displacement Dx_1 from t = 0 to t = 60 s and the displacement Dx_2 from t = 60 s to $t \to \infty$.

1. The velocity of the boat is constant during the first 60 seconds; thus, the displacement is simply the velocity times the elapsed time :

   Dx_1 =uDt = (8 m/s) (60 s) = 480 m

2. The remaining displacement is given by the integral of the velocity from t = 60 s to $t = \infty$. We use Equation to calculate the integral.

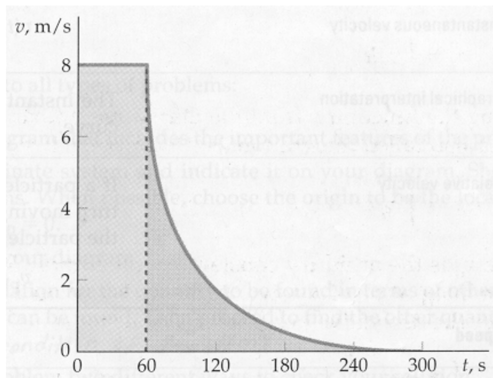
$$Dx_2 = \int_{60s}^{\infty} u\,dt = \int_{60s}^{\infty} \frac{u_0 t_1^2}{t^2}\,dt = u_0 t_1^2 \int_{60s}^{\infty} t^{-2}\,dt = u_0 t_1^2 \left.\frac{t^{-1}}{-1}\right|_{60s}^{\infty}$$

$$= \frac{u_0 t_1^2}{60s} = \frac{(8\,m/s)\,(60s)^2}{60s} = 480\ \text{m}$$

3. The total displacement is the sum of the displacements found above :

$Dx = Dx_1 + Dx_2 = 480\text{ m} + 480\text{ m} = 960\text{ m}.$

Remark : Note that the area under the u-verses-t curve is finite. Thus, even though the boat never stops moving, it travels only a finite distance. A better representation of the velocity of a coasting boat might be an exponentially decreasing function. In that case, the boat would also coast a finite distance in the interval $60\text{ s} \le t < \infty$



Q.2 The acceleration of a particle moving in a straight line varies with its displacement as :

$$a = 2s$$

velocity of the particle is zero at zero displacement. Find the corresponding velocity displacement equation.

Sol. $a = 2s$

or $v\dfrac{dv}{ds} = 2s$ or $v\,dv = 2s\,ds$ or $\int_0^v v\,dv = \int_0^s 2s\,ds$

or $\left[\dfrac{v^2}{2}\right]_0^v = \left[s^2\right]_0^s$ or $\dfrac{v^2}{2} = s^2$

Hence, $v = \pm\sqrt{2}\,s$

(This is the desired velocity-displacement equation)

Q.3 The retardation of a particle moving in a straight line is proportional to it's displacement (proportionality constant being unity). Find the total distance covered by the particle till it comes to rest. Given that velocity of particle is v_0 at zero displacement.

Sol. Given $a = -s$ or $v\dfrac{dv}{ds} = -s$

Hence, $\int_{v_0}^{0} v\,dv = -\int_0^s s\,ds$

$\therefore \qquad -\frac{v_0^2}{2} = -\frac{s^2}{2} \qquad$ or $s = v_0$

NOTE :

Students often remain confused that the equation $s = v_0$ is not dimensionally correct. They actually forget writing dimensions of proportionality constant which is unity here.

Q.4 The motion of a body is given by the equation

$$\frac{dv(t)}{dt} = 6.0 - 3v\ (t)$$

when v (t) is the speed in m/s and t in sec. If the body was at rest at t = 0 ;

Then, test the correctness of the following results.

(a) the terminal speed is 2.0 m/s.

(b) the magnitude of initial acceleration is 6.0 m/s^2.

(c) the speed varies with time as $v(t) = 2\ (1 - e^{-3t})$ m/s.

(d) the speed is 1.0 m/s when the acceleration is half the initial value.

Sol. Given equation is $\frac{dv(t)}{dt} = 6.0 - 3\ v\ (t)$ .... (1)

(a) At terminal speed, acceleration $\frac{dv(t)}{dt} = 0$

$\therefore \quad 6 - 3v = 0 \qquad$ or $\qquad v = \frac{6}{3} = 2.0$ m/s

(b) For initial acceleration (t = 0) ;

$\therefore \quad$ initial acceleration, $a_0 = \frac{dv(0)}{dt} = 6.0$ m/s^2

(c) Equation (1) may be expressed as

$\frac{dv}{6-3v} = dt$. Integrating $\frac{\log_e(6-3v)}{-3} = t + C_1$ .... (2)

where C_1 is a constant of integration .

Given at t = 0, v = 0

$\therefore \quad$ equation (2) gives $\frac{\log_e 6}{-3} = C_1$

$\therefore \quad$ equation (2) becomes $\frac{\log_e(6-3v)}{-3} = t + \frac{\log_e 6}{-3}$

$\therefore \quad \log_e(6 - 3v) - \log_e 6 = -3t \qquad \Rightarrow \qquad \log_e\left\{\frac{(6-3v)}{6}\right\} = -3t$

or $\left(1 - \frac{v}{2}\right) = e^{-3t} \qquad \Rightarrow \qquad v = 2\ (1 - e^{-3t})$ m/s

(d) When $a = \frac{dv}{dt} = \frac{a_0}{2} = 3$ m/s^2. We have from (1), $3 = 6.0 - 3\ v\ (t)$

This gives, v (t) = 1 m/s i.e. all alternatives (a), (b), (c) , (d) are correct.

PRACTICE QUESTIONS BASED ON MOTION DEFINING PARAMETERS

DISTANCE & DISPLACEMENT :

Q.1 The location of a particle is changed. What can we say about the displacement and distance covered by the particle ?

(A) Both cannot be zero.

(B) One of the two may be zero.

(C) Both must be zero.

(D) If one is positive, the other is negative and vice-versa.

Q.2 A body covered a distance of 5m along a semicircular path. The ratio of distance to displacement is :

(A) 11 : 7 (B) 12 : 5 (C) 8 : 3 (D) 7 : 5

SPEED AND VELOCITY :

Q.3 The numerical value of the ratio of velocity to speed is :

(A) Always less than one (B) Always equal to one

(C) Always more than one (D) Equal to or less than one

Q.4 The magnitude of average velocity is equal to the average speed, when a particle moves :

(A) On a curved path (B) In the same direction

(C) With constant acceleration (D) With constant retardation.

Q.5 A body covers one-third of the distance with a velocity v_1, the second one-third of the distance with a velocity v_2 and the remaining distance with a velocity v_3. The average velocity is :

(A) $\frac{v_1 + v_2 + v_3}{3}$ (B) $\frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$ (C) $\frac{v_1v_2 + v_2v_3 + v_3v_1}{3}$ (D) $\frac{v_1\ v_2\ v_3}{3}$

Q.6 A particle moving in a straight line covers half the distance with speed of 3m/sec. The other half of the distance is covered in two equal time intervals with speed of 4.5 m/sec and 7.5 m/sec respectively. The average speed of the particle during this motion is :

(A) 4.0 m/sec (B) 5.0 m/sec (C) 5.5 m/sec (D) 4.8 m/sec

Q.7 The displacement of a body is given by $4s = M + 2Nt^4$, where M & N are constants. The velocity of the body at any instant is :

(A) $\frac{M + 2Nt^4}{4}$ (B) 2N (C) $\frac{M + 2N}{4}$ (D) $2Nt^3$

Q.8 The displacement of a body at any time t after starting is given by $s = 10t - \frac{1}{2}(0.2)\ t^2$. The velocity of the body is zero after :

(A) 50 sec (B) 100 sec (C) 80 sec (D) 40 sec

Q.9 Three persons are initially at the three corners of an equilateral triangle whose side is equal to d. Each person now moves with a uniform speed v in such a way that the first moves directly towards the second, the second directly towards the third, the third directly towards the first. The three persons will meet after a time equal to-

(A) $\frac{2d}{3v}$ seconds (B) $\frac{d}{v}$ seconds (C) $\frac{2d}{\sqrt{3}v}$ seconds (D) $\frac{d}{\sqrt{3}v}$ seconds

UNIFORM ACCELERATION :

Q.10 The displacement s of a particle is proportional to the first power of time t, i.e. $s \propto t$; then the acceleration of the particle is-

(A) Infinite (B) Zero (C) A small finite value (D) A large finite value

Q.11 The displacement s of a particle is proportional to the second power of time t, i.e. $s \propto t^2$; then the initial velocity of the particle is :

(A) Infinite (B) Zero (C) Positive finite value (D) Negative finite value

Q.12 A particle experiences constant acceleration for 20 seconds after starting from rest. If it travels a distance s_1 in the first 10 seconds and distance s_2 in the next 10 seconds, then-

(A) $s_2 = s_1$ (B) $s_2 = 2s_1$ (C) $s_2 = 3s_1$ (D) $s_2 = 4s_1$

Q.13 A point moves with uniform acceleration and v_1, v_2 & v_3 denote the average velocities in the three successive intervals of time t_1, t_2 & t_3. Which of the following relations is correct ?

(A) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$ (B) $(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$

(C) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_1 - t_3)$ (D) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 - t_3)$

Q.14 If a body starts from rest, the time in which it covers a particular displacement with uniform acceleration is :

(A) Inversely proportional to the square root of the displacement.

(B) Inversely proportional to the displacement.

(C) Directly proportional to the displacement.

(D) Directly proportional to the square root of displacement.

Q.15 For a moving body, at any instant of time :

(A) If the body is not moving, the acceleration is necessarily zero.

(B) If the body is slowing, the acceleration is negative.

(C) If the body is slowing, the distance is negative.

(D) If the distance, velocity and acceleration at that instant are known, we can find the displacement at any given time in future.

NON-UNIFORM ACCELERATION :

Q.16 The distance travelled by a particle is directly proportional to $t^{1/2}$, where t = time elapsed. What is the nature of motion ?

(A) Increasing acceleration
(B) Decreasing acceleration
(C) Increasing retardation
(D) Decreasing retardation

Q.17 A particle moves along a straight line such that it's displacement at any time t is given by :

$$(s = t^3 - 6t^2 + 3t + 4) \text{ metres.}$$

The velocity, when the acceleration is zero is :

(A) $3 m\ sec^{-1}$
(B) $-12 m\ sec^{-1}$
(C) $42 m\ sec^{-1}$
(D) $-9\ m\ sec^{-1}$.

ANSWERS :

1. (A)	**2.** (A)	**3.** (D)	**4.** (B)	**5.** (B)	**6.** (A)
7. (D)	**8.** (A)	**9.** (A)	**10.** (B)	**11.** (B)	**12.** (C)
13. (B)	**14.** (D)	**15.** (B)	**16.** (D)	**17.** (D)	

LEVEL # 1

DISTANCE & DISPLACEMENT

Q.1 A body moves 6m north, 8m east and 10m vertically upwards. What is its resultant displacement from initial position ?

(A) $10\sqrt{2}$ m (B) 10m (C) $\frac{10}{\sqrt{2}}$ m (D) 10×2m

Q.2 A man goes 10m towards North, then 20m towards east then displacement is-

(A) 22.5 m (B) 25 m (C) 25.5m (D) 30m

Q.3 A person moves 30m north and then 20m towards east and finally $30\sqrt{2}$ m in south-west direction. The displacement of the person from the origin will be-

(A) 10m along north (B) 10m along south (C) 10m along west (D) Zero

Q.4 An aeroplane flies 400 m north and 300 m south and then flies 1200 m upwards then net displacement is-

(A) 1200 m (B) 1300 m (C) 1400 m (D) 1500 m

Q.5 An athlete completes one round of a circular track of radius R in 40 sec. What will be his displacement at the end of 2 min. 20 sec ?

(A) Zero (B) 2R (C) $2\pi R$ (D) $7\pi R$

Q.6 A wheel of radius 1 meter rolls forward half a revolution on a horizontal ground. The magnitude of the displacement of the point of the wheel initially in contact with the ground is-

(A) 2π (B) $\sqrt{2}\,\pi$ (C) $\sqrt{\pi^2 + 4}$ (D) π

UNIFORM MOTION

Q.7 A person travels along a straight road for half the distance with velocity v_1 and the remaining half distance with velocity v_2. The average velocity is given by-

(A) $v_1 \; v_2$ (B) $\frac{v_2^2}{v_1^2}$ (C) $\frac{v_1 + v_2}{2}$ (D) $\frac{2v_1v_2}{v_1 + v_2}$

Q.8 The displacement time graph for two particles A and B are straight lines inclined at angles of 30º and 60º with the time axis. The ratio of velocities i.e. $V_A : V_B$ is-

(A) 1 : 2 (B) 1 : $\sqrt{3}$ (C) $\sqrt{3}$: 1 (D) 1 : 3

Q.9 A car travels from A to B at a speed of 20 km/hr and returns at a speed of 30 km/hr. The average speed of the car for the whole journey is-

(A) 25 km/hr (B) 24 km/hr (C) 50 km/hr (D) 5 km/hr

MOTION DEFINING PARAMETERS

Q.10 A boy walks to his school at a distance of 6 km with constant speed of 2.5 km/hour and walks back with a constant speed of 4 km/hr. His average speed for round trip expressed in km/hour is-

(A) 24/13 (B) 40/13 (C) 3 (D) 1/2

Q.11 A car travels the first half of a distance between two places at a speed of 30 km/hr and the second half of the distance at 50 km/hr. The average speed of the car for the whole journey is-

(A) 42.5 km/hr (B) 40.0 km/hr (C) 37.5 km/hr (D) 35.0 km/hr

Q.12 One car moving on a straight road covers one third of the distance with 20 km/hr and the rest with 60 km/hr. The average speed is-

(A) 40 km/hr (B) 80 km/hr (C) $46\frac{2}{3}$ km/hr (D) 36 km/hr

Q.13 A car moves for half of its time at 80 km/h and for rest half of time at 40 km/h. Total distance covered is 60 km. What is the average speed of the car ?

(A) 60 km/h (B) 80 km/h (C) 120 km/h (D) 180 km/h

Q.14 A train has a speed of 60 km/h for the first one hour and 40 km/h for the next half hour. Its average speed in km/h is-

(A) 50 (B) 53.33 (C) 48 (D) 70

Q.15 Which of the following is a one dimensional motion-

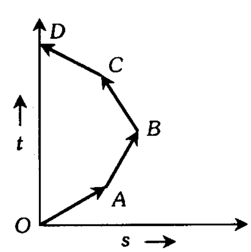
(A) Landing of an aircraft (B) Earth revolving around the sun

(C) Motion of wheels of a moving train (D) Train running on a straight track

Q.16 A 150 m long train is moving with a uniform velocity of 45 km/h. The time taken by the train to cross a bridge of length 850 meters is-

(A) 56 sec (B) 68 sec (C) 80 sec (D) 92 sec

Q.17 Which of the following options is correct for the object having a straight line motion represented by the following graph



(A) The object moves with constantly increasing velocity from O to A and then it moves with constant velocity.

(B) Velocity of the object increases uniformly

(C) Average velocity is zero.

(D) The graph shown is impossible.

MOTION DEFINING PARAMETERS

Q.18 The numerical ratio of displacement to the distance covered is always-

(A) Less than one (B) Equal to one

(C) Equal to or less than one (D) Equal to or greater than one

Q.19 A 100 m long train is moving with a uniform velocity of 45 km/hr. The time taken by the train to cross a bridge of length 1 km is-

(A) 58 s (B) 68 s (C) 78 s (D) 88 s

Q.20 A particle moves for 20 seconds with velocity 3 m/s and then velocity 4 m/s for another 20 seconds and finally moves with velocity 5 m/s for next 20 seconds. What is the average velocity of the particle ?

(A) 3 m/s (B) 4 m/s (C) 5 m/s (D) Zero

Q.21 The correct statement from the following is-

(A) A body having zero velocity will not necessarily have zero acceleration.

(B) A body having zero velocity will necessarily have zero acceleration.

(C) A body having uniform speed can have only uniform acceleration.

(D) A body having non-uniform velocity will have zero acceleration.

Q.22 A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further will it penetrate before coming to rest assuming that it faces constant resistance to motion ?

(A) 1.5 cm (B) 1.0 cm (C) 3.0 cm (D) 2.0 cm

Q.23 Two boys are standing at the ends A and B of a ground where AB = a. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other boy in a time t, where t is-

(A) $\frac{a}{\sqrt{v^2 + v_1^2}}$ (B) $\sqrt{\frac{a^2}{(v^2 - v_1^2)}}$ (C) $\frac{a}{(v - v_1)}$ (D) $\frac{a}{(v + v_1)}$

Q.24 A car travels half the distance with constant velocity of 40 kmph and the remaining half with a constant velocity of 60 kmph. The average velocity of the car in kmph is-

(A) 40 (B) 45 (C) 48 (D) 50

LEVEL # 2

MORE THAN ONE CHOICE MAY BE CORRECT :

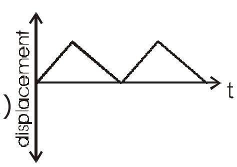
DISTANCE - DISPLACEMENT

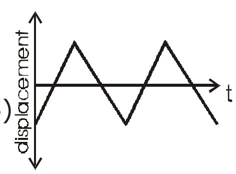
Q. 1 A body travels 4m north and then turns east and travels 3m due east. The body has-

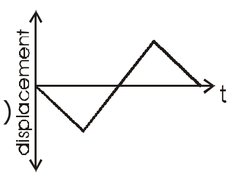
(A) Displacement 5 m (B) Displacement 7m

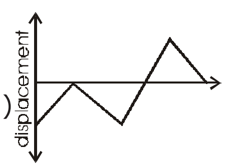
(C) Distance covered 5 m (D) Distance covered 7 m

Q.2 Three stations Ratlam-kota-Delhi lie on single line and we consider Kota as origin. If a train continuously makes to and fro trips starting from Ratlam to kota to Delhi to kota to Ratlam and so on, then most appropriate displacement time graph for two trips will be.

(A) 

(B) 

(C) 

(D) 

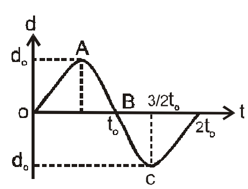
Q.3 A body covered a distance of L m along a curved path of a quarter circle. The ratio of distance to displacement is -

(A) $\pi / 2\sqrt{2}$ (B) $2\sqrt{2} / \pi$ (C) $\pi / \sqrt{2}$ (D) $\sqrt{2} / \pi$

Q.4 A particle is moving along trajectory, $y = \frac{2}{3}(x-1)^{3/2}$. The displacement and distance for x = 1 to 4 will be respectively.

(A) $\frac{2}{3}\sqrt{27}$,3 (B) $\sqrt{21}, \frac{16}{3}$ (C) $\sqrt{21}, \frac{14}{3}$ (D) $\frac{2}{3}\sqrt{27}, \frac{14}{3}$

<u>Question 5, 6, 7 are based on following graph</u>



Q.5 This graph is possible if d represents

(A) distance (B) displacement

(C) possible for both (D) possible for none

Q.6 If y axis of above graph represents displacement of a one dimensional motion then find (distance , displacement) in time t_o, $\frac{3}{2}t_o$, $2t_o$:

(A) (3d,0), (3d, -d), (4d,0) (B) (2d,0), (2d, -d), (4d,0)

(C) (2d,0), (3d, -d), (4d,0) (D) (2d,0), (3d, -d), (0,0)

MOTION DEFINING PARAMETERS

Q.7 If it is 2-D motion then is it possible to find answers of above question ?
(A) Absolutely yes (B) Absolutely no (C) Partially yes (D) Partially no

Q.8 Distance travelled by a particle which is stuck from road to wheel of radius R after the completion of half revolution is :
(A) $2\pi R$ (B) πR (C) 4R (D) $R\sqrt{\pi^2+4}$

hint : $x = R(\theta - \sin\theta)$, $y = R(1-\cos\theta)$

Q.9 Choose the incorrect option/s.
(A) Magnitude of displacement can be equal to distance travelled.
(B) Distance cannot decrease with time for a moving body.
(C) If the length of position vector for two different positions are same then there is surely no displacement
(D) Nothing is in the state of absolute rest or absolute motion.

Q.10 What is the displacement of the point of the wheel initially in contact with the ground when the wheel rolls forward half a revolution ? Take the radius of the wheel R and x-axis in forward direction.
(A) $\frac{R}{\sqrt{\pi^2+4}}$ (B) $R\sqrt{\pi^2+4}$ (C) $2\pi R$ (D) πR

Q.11 An electrician climbs up an electric pole. In each attempt, he moves up by 50 cm but slips back by 20 cm. Moving up step takes 2 sec and slipping takes 1 sec. The total length of path and time taken to climb the 4.5 m pole is
(A) 4.5 m, 18 sec (B) 10.5m, 45 sec (C) 4.5m, 45 sec (D) 10.5m, 18sec

SPEED AND VELOCITY

Q.12 If the velocity of a body is constant-
(A) |Velocity| = speed (B) |Average velocity| = speed
(C) Velocity = average velocity (D) Speed = average speed

Q.13 A body moves in a straight line along x-axis. Its distance x (in metre) from the origin is given by $x = 8t - 3t^2$. The average speed in the interval t = 0 to t = 1 second is-
(A) 5 ms^{-1} (B) -4 ms^{-1} (C) 6 ms^{-1} (D) zero

Q.14 If a particle travels a linear distance at speed v_1 and comes back along the same track at speed v_2 -
(A) Its average speed is arithmetic mean $(v_1 + v_2)/2$
(B) Its average speed is geometric mean $\sqrt{v_1 v_2}$
(C) Its average speed is harmonic mean $2v_1v_2 / (v_1 + v_2)$
(D) Its average velocity is zero

Q.15 If the displacement of a particle varies with time according to the relation $x = \frac{K}{b}[1-\exp(-bt)]$, then the velocity (V) of the particle is

(A) $V = K \exp(-bt)$ (B) $V = \frac{K}{b} \exp(-bt)$

(C) $V = \frac{K^2}{b} \exp(-bt)$ (D) $V = \frac{K}{b^2} \exp(-bt)$

Q.16 The instantaneous velocity v of a particle is related to its displacement x according to the relation $v = ax + b$, where $a > 0$ and $b \leq a/7$. Which of the following statement (s) is (are) true if x = 0 at t = 0.

(A) The displacement of the particle at time t is $x = \frac{b}{a}(e^{at} - 1)$

(B) The particle will experience a retardation if b < 0

(C) The particle will be at rest at time t = 0

(D) The acceleration of the particle is constant.

Q.17 Three particles start their motion from the origin at the same time. The first moves with a velocity u_1 along the x-axis, the second moves along the y-axis with a velocity u_2 and the third along the straight line y = x. Then, the velocity of the third particle so that all the three may always lie on the same line is

(A) $\frac{u_1 + u_2}{2}$ (B) $\sqrt{u_1u_2}$ (C) $\frac{u_1u_2}{u_1 + u_2}$ (D) $\frac{\sqrt{2}u_1u_2}{u_1 + u_2}$

INSTANTANEOUS VELOCITY

Q.18 On a displacement-time graph two straight lines make angles 30º and 60º with the time-axis. The ratio of the velocities represented by them is

(A) $1 : \sqrt{3}$ (B) 1 : 3 (C) 3 : 1 (D) $\sqrt{3} : 1$

Q.19 A particle is moving along X-axis, its position varying with time as $x(t) = 2t^3 - 3t^2 + 1$

(a) At what time instants, is its velocity zero.

(b) What is the velocity when it passes through origin ?

(A) t = 0, t = 1, v = 1 (B) t = 0, t = 2, v = 0

(C) t = 0, t = 1, v = 0 (D) t = 0, t = 1, v = 2

AVERAGE VELOCITY

Q.20 A particle moving along a straight line moves with a uniform velocity v_1 for some time and with uniform velocity v_2 for the next equal time. The average velocity v for the total time of travel is given by

(A) $v = \frac{v_1 + v_2}{2}$ (B) $v = \sqrt{v_1 v_2}$ (C) $v = \frac{v_1 v_2}{v_1 + v_2}$ (D) $v = \frac{2v_1 v_2}{v_1 + v_2}$

Q.21 A particle moving along a straight line moves with a uniform velocity v_1 for a distance x and with a uniform velocity v_2 for the next equal distance. The average velocity v is given by

(A) $v = \frac{v_1 + v_2}{2}$ (B) $v = \sqrt{v_1 v_2}$ (C) $v = \frac{v_1 v_2}{v_1 + v_2}$ (D) $v = \frac{2v_1 v_2}{v_1 + v_2}$

Q.22 A particle travels half the distance with velocity u. The remaining part of the distance is covered with velocity v_1 for the first half time and v_2 for the remaining half time. Find the average velocity of the particle during the complete motion.

(A) $\frac{2u(v_1 - v_2)}{2u + v_1 + v_2}$ (B) $\frac{2u(v_1 + v_2)}{2u - v_1 - v_2}$ (C) $\frac{2u(v_1 - v_2)}{2u - v_1 - v_2}$ (D) $\frac{2u(v_1 + v_2)}{2u + v_1 + v_2}$

AVERAGE ACCELERATION

Q.23 A particle is moving eastward with a velocity of 5 ms^{-1}. In 10s the velocity changes to 5ms^{-1} northward. The average acceleration in this time is-

(A) zero
(B) $1/\sqrt{2}$ ms^{-2} towards north-west
(C) $1/\sqrt{2}$ ms^{-2} towards north-east
(D) 1/2 ms^{-2} towards north-west

UNIFORM ACCELERATION

Q.24 Let $\vec{\upsilon}$ and $\vec{a}$ denote the velocity and acceleration respectively of a body in one-dimensional motion.

(A) $|\vec{\upsilon}|$ must decrease when $\vec{a} < 0$.
(B) Speed must increase when $\vec{a} > 0$.
(C) Speed will increase when both $\vec{\upsilon}$ and $\vec{a}$ are <0.
(D) Speed will decrease when $\vec{\upsilon} < 0$ and $\vec{a} > 0$.

Q.25 A moving body may have
(A) varying speed without having varying velocity
(B) varying velocity without having varying speed
(C) nonzero acceleration without having varying velocity
(D) nonzero acceleration without having varying speed.

Q.26 Mark the incorrect statement (s) for a particle moving along a straight line:

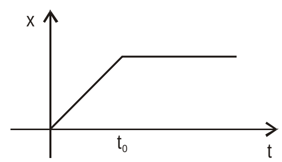
(A) If the velocity and acceleration have opposite sign, the particle is slowing down.

(B) If the position vector and velocity vector have opposite sign, the particle is moving towards the origin.

(C) If the velocity is zero at an instant, the acceleration should also be zero at that instant.

(D) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval

Q.27 Figure shows the displacement - time graph of a particle moving on the X-axis.



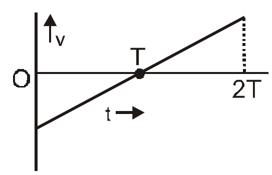
(A) The particle is continuously moving along the positive x direction

(B) The particle remains at rest

(C) The velocity increases up to a time t_0 and then becomes constant

(D) The particle moves at a constant velocity up to a time t_0 and then stops.

Q.28 The figure shows the velocity (υ) of a particle plotted against time (t).



(A) The particle changes its direction of motion at some point.

(B) The acceleration of the particle remains constant.

(C) The displacement of the particle is zero.

(D) The initial and final speeds of the particle are the same

Q.29 A particle starts from the origin of coordinates at time t = 0 and moves in the xy plane with a constant acceleration α in the y-direction. Its equation of motion is $y=\beta x^2$. Its velocity component in the x-direction is

(A) variable (B) $\frac{\sqrt{2\alpha}}{\beta}$ (C) $\frac{\alpha}{2\beta}$ (D) $\frac{\sqrt{\alpha}}{2\beta}$

Q.30 Two particles A and B start simultaneously from the same point and move in a horizontal plane. A has an initial velocity u_1 due east and acceleration a_1 due north. B has an initial velocity u_2 due north and acceleration a_2 due east.

(A) Their paths must intersect at some point.

(B) They must collide at some point.

(C) They will collide only if $a_1u_1 = a_2u_2$.

(D) If $u_1 > u_2$ and $a_1 < a_2$, the particles will have the same speed at some point of time.

NON-UNIFORM ACCELERATION

Q.31 The displacement of particle is zero at t = 0 and at t = t, it is x. It starts moving in the x direction with velocity, which varies as $v = k\sqrt{x}$, where k is constant. The velocity-

(A) varies with time
(B) independent to time
(C) inversely proportional to time
(D) nothing can be said

Q.32 A particle moves along the x-axis as follows: it starts from rest at t = 0 from a point x = 0 and comes to rest at t = 1 at a point x = 1. No other information is available about its motion for the intermediate time (0<t<1). If α denotes the instantaneous acceleration of the particle then

(A) α cannot remain positive for all t in the interval $0 \leq t \leq 1$
(B) $|\alpha|$ cannot exceed 2 at any point in its path
(C) $|\alpha|$ must be ≥ 4 at some point or points in its path
(D) α must change sign during the motion, but no other assertion can be made with the information given

Q.33 The displacement (x) of a particle depends on time (t) as

$x = \alpha t^2 - \beta t^3$.

(A) The particle will return to its starting point after time α/β.
(B) The particle will come to rest after time $2\alpha/3\beta$.
(C) The initial velocity of the particle was zero but its initial acceleration was not zero.
(D) No net force will act on the particle at $t = \alpha/3\beta$.

Q.34 A particle moves with an initial velocity υ_o and retardation $\alpha\upsilon$, where υ is its velocity at any time t.

(A) The particle will cover a total distance υ_o/α.
(B) The particle will come to rest after time $1/\alpha$.
(C) The particle will continue to move for a very long time.
(D) The velocity of the particle will become $\upsilon_o/2$ after time $1/\alpha$.

ANSWER KEY

LEVEL # 1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	A	C	A	B	C	D	D	B	B	C	D	A	B	D
Que.	16	17	18	19	20	21	22	23	24						
Ans.	C	C	C	D	B	A	B	B	C						

LEVEL # 2

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A,D	B	A	C	B	C	B	C	C	B	B	All	A	C,D	A
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	A,B	C	B	C	A	D	D	B	C,D	B,D	C	D	All	D	A,C,D
Que.	31	32	33	34											
Ans.	A	A,C	All	A,C											