

ASSIGNMENT

Q.1 Solve each of the following equations for $x \in \mathbb{C}$.

(a) $(x^2 - 5x + 7)^2 - (x - 2)(x - 3) = 1$ (b) $(5 + 2\sqrt{6})^{x^2 - 3} + (5 - 2\sqrt{6})^{x^2 - 3} = 10$

(c) $\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}} \dots \infty$

(d) $5x^{1/2n} - 3x^{1/n} = 2$

(e) $2^{x^2} : 4^x = 8 : 1$.

(f) $ix^2 - 2x + i = 0$

(g) $(2 + i)x^2 + (i - 5)x + 2(1 - i) = 0$

Q.2 Show that the roots of the equation $x^2 + 2(bm + 5)x + 2(9m^2 + 25) = 0$ are complex unless $m = \frac{5}{3}$.

Q.3 Show that the roots of the equation $(x - a)(x - b) = abx^2$ $a, b, \in \mathbb{R}$ are always real. Why they are equal.

Q.4 Prove that the roots of the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ are imaginary unless $ad = bc$.

Q.5 If α, β are roots of the equation $ax^2 + bx + c = 0$ find the value of :

(a) $\alpha^2 + \beta^2$

(b) $\alpha^6 + \beta^6$

(c) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(d) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Q.6 Form a quadratic equation whose roots are given as

(a) $\sqrt{3}, \frac{1}{\sqrt{3}}$

(b) $\frac{m-n}{m+n}, \frac{m+n}{m-n}$.

Q.7 Form a quadratic equation with rational coefficients one of whose roots :

(a) $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$

(b) $-2 - \sqrt{-5}$

Q.8 If k is the ratio of the roots of the equation $x^2 - px + q = 0$, prove that $\frac{k^2 + 1}{k} = \frac{p^2 - 2q}{q}$.

Q.9 If one roots of the equation $ax^2 + bx + c = 0$ is triple of the other, show that $3b^2 = 16ac$.

- Q.10** For what value of m will be the equation $\frac{x^2 - bx}{ax - c} = \frac{m-1}{m+1}$ have roots equal in magnitude but opposite in sign ?
- Q.11** If the sum of the roots of the equation $x^2 - mx + n = 0$ be p times their difference, then show that $m^2(p^2 - 1) = 4p^2n$.
- Q.12** The coefficient of x in the equation $x^2 + px + q = 0$ was taken as 17 in place of 13 and thus its roots were found to be -2 and -15 . Find the roots of the original equation.

ANSWER KEY

- 1.** (a) $2, 3, \frac{5 \pm \sqrt{3}i}{2}$ (b) $\pm 2, \pm \sqrt{2}$ (c) 3 (d) $1, \left(\frac{2}{3}\right)^{2n}$
- (e) $-1, 3$ (f) $(-1 \pm \sqrt{2})i$ (g) $1 - i, \frac{4 - 2i}{5}$
- 5.** (a) $\frac{b^2 - 2ac}{a^2}$ (b) $\frac{9a^2b^2c^2 + b^6 - 6ab^4c - 2a^3c^3}{a^6}$
- (c) $\frac{b^2 - 2ac}{ac}$ (d) $\frac{3abc - b^3}{a^2c}$
- 6.** (a) $\sqrt{3}x^2 - 4x + \sqrt{3} = 0$
 (b) $(m^2 - n^2)x^2 - 2(m^2 + n^2)x + (m^2 - n^2) = 0$
- 7.** (a) $x^2 - 4x + 1 = 0$ (b) $x^2 + 4x + 9 = 0$
- 10.** $\frac{a-b}{a+b}$
- 12.** $-3, -10$