Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

VERY SHORT ANSWER TYPE QUESTIONS :

VSA.1 Solve the equation : $(x^2 - 5x + 7)^2 - (x - 2)(x - 3) = 1$

Sol. We have,

$(x^2 - 5x + 7)^2 - (x - 2)(x - 3) = 1$

$(x^2 - 5x + 7)^2 - (x^2 - 5x + 6) = 1$

Put, $x^2 - 5x + 6 = y$ ....(1)

$\Rightarrow (y + 1)^2 - y = 1$

$\Rightarrow y^2 + 2y + 1 - y = 1$

$\Rightarrow y^2 + y = 0$

$\Rightarrow y(1 + y) = 0$

$\Rightarrow y = 0, y = -1.$

Put in (1)

$\therefore y = 0$

$x^2 - 5x + 6 = 0$

$\Rightarrow (x - 2)(x - 3) = 0$

$\Rightarrow x = 2, 3$

$y = -1$

$x^2 - 5x + 6 = -1$

$\Rightarrow x^2 - 5x + 7 = 0$

$\Rightarrow x = \dfrac{5 \pm \sqrt{25-28}}{2} = \dfrac{5 \pm \sqrt{3}i}{2}.$

VSA.2 Solve the equation : $\sqrt{3x+1} - \sqrt{x-1} = 2$

Sol. We have,

$\sqrt{3x+1} - \sqrt{x-1} = 2$ [squaring both sides]

$\Rightarrow 3x + 1 + x - 1 - 2\sqrt{(3x-1)(x-1)} = 4$

$\Rightarrow 4x - 4 = 2\sqrt{3x^2 - 2x - 1}$

$\Rightarrow 2(x - 1) = \sqrt{3x^2 - 2x - 1}$ [Again squaring]

$\Rightarrow 2^2(x - 1)^2 = 3x^2 - 2x - 1$

$\Rightarrow 4(x - 1)^2 = 3x^2 - 2x - 1$

$\Rightarrow x^2 - 6x + 5 = 0$

$\Rightarrow (x - 1)(x - 2) = 0$

$\Rightarrow x = 1$ or $x = 5.$

VSA.3 Solve the equation : $\sqrt{x+1} + \sqrt{x-1} = 0$

Sol. $\sqrt{x+1} + \sqrt{x-1} = 0$

$\Rightarrow \sqrt{x+1} = -\sqrt{x-1}$ [Squaring]

$\Rightarrow x + 1 = x - 1$

$\Rightarrow 1 \neq -1$

$\Rightarrow$ No solution.

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

VSA.4 Solve the equation : $5^{x^2 + 3x} = \frac{1}{25}$.

Sol. The given equation is

$$5^{x^2 + 3x} = \frac{1}{25}$$

$\Rightarrow$ $5^{x^2 + 3x} = 5^{-2}$

$\Rightarrow$ $x^2 + 3x = -2$

$\Rightarrow$ $x^2 + 3x + 2 = 0$

$\Rightarrow$ $(x + 1)(x + 2) = 0$

$\Rightarrow$ $x = -1, -2$.

Hence, the roots of the given equation are –1, –2.

VSA.5 Solve the equation : $\sqrt{13-3x} = x - 3$.

Sol. We have,

$$\sqrt{13-3x} = x - 3$$

$\Rightarrow$ $13 - 3x = (x - 3)^2$ [On squaring both sides]

$\Rightarrow$ $x^2 - 3x - 4 = 0$

$\Rightarrow$ $(x - 4)(x + 1) = 0$

$\Rightarrow$ $x = -1, 4$

We observe that x = 4 satisfies the given equation but x = –1 does not satisfy it. So, x = 4 is the solution of the given equation.

VSA.6 Solve the equations :

(a) $x^{2/3} + x^{1/3} - 2 = 0$ (b) $(x^2 - 3x)^2 + 3(x^2 - 3x) + 2 = 0$

Sol.

(a) We have,

$x^{2/3} + x^{1/3} - 2 = 0$ ......(1)

$\Rightarrow$ $x^{(1/3)^2} + x^{1/3} - 2 = 0$

Let $y = x^{1/3}$

$\therefore$ (1) $y^2 + y - 2 = 0$

$$y = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = 1, -2.$$

Either $y = 1$ or $y = -2$

$\therefore$ $x^{1/3} = 1$ $\therefore$ $x^{1/3} = -2$

$\Rightarrow$ $x = (1)^3 = 1$ $\Rightarrow$ $x = (-2)^3 = -8$

$\therefore$ The roots are 1, –8.

(b) We have,

$(x^2 - 3x)^2 + 3(x^2 - 3x) + 2 = 0$ ......(1)

Let $y = x^2 - 3x$

$\therefore$ (1) $y^2 + 3y + 2 = 0$

$$y = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2} = -1, -2.$$

Either $y = -1$

$\therefore$ $x^2 - 3x = -1$

$\Rightarrow$ $x^2 - 3x + 1 = 0$

$\Rightarrow$ $x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$

or $y = -2$

$\therefore$ $x^2 - 3x = -2$

$\Rightarrow$ $x^2 - 3x + 2 = 0$

$\Rightarrow$ $x = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} = 2, 1$

$\therefore$ The roots are $\frac{3 \pm \sqrt{5}}{2}$, 2, 1.

VSA.7 Solve the equations : $5^{2x} - 5^{x+3} + 125 = 5^x$.

Sol. We have,

$5^{2x} - 5^{x+3} + 125 = 5^x$

$\Rightarrow$ $(5^x)^2 - 5^x . 5^3 + 125 - 5^x = 0$

$\Rightarrow$ $(5^x)^2 - 5^x (5^3 + 1) + 125 = 0$

$\Rightarrow$ $(5^x)^2 - 126 . 5^x + 125 = 0$

Let $y = 5^x$

$\therefore$ (1) $y^2 - 126y + 125 = 0$

$$\therefore \quad y = \frac{126 \pm \sqrt{(-126)^2 - 4(1)(125)}}{2(1)} = 1,125$$

Either $y = 1$

$\Rightarrow$ $5^x = 1$

$\Rightarrow$ $5^x = 5^0$

$\Rightarrow$ $x = 0$

or $y = 125$

$\Rightarrow$ $5^x = 125$

$\Rightarrow$ $5^x = 5^3$

$\Rightarrow$ $x = 3$.

$\therefore$ The roots are 0, 3.

VSA.8 Evaluate : $\sqrt{6 + \sqrt{6 + \sqrt{6 + ...\infty}}}$.

Sol. Let $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + ...\infty}}}$.

Then, $x = \sqrt{6 + x}$ ....(1)

Squaring both sides of (1), we get

$x^2 = 6 + x$

$\Rightarrow$ $x^2 - x - 6 = 0$

$\Rightarrow \quad x^2 - 3x + 2x - 6 = 0$

$\Rightarrow \quad x(x - 3) + 2(x - 3) = 0$

$\Rightarrow \quad (x - 3)(x + 2) = 0$

Either $\quad x - 3 = 0$ or $x + 2 = 0$

$\Rightarrow \quad x = 3$ or $x = -2$.

Since, x cannot be negative. Therefore, the value of the given expression is 3.

VSA.9 Solve the equation : $(x^2 - 3x)^2 - 5(x^2 - 3x) + 6 = 0$.

Sol. We have,

$(x^2 - 3x)^2 - 5(x^2 - 3x) + 6 = 0 \quad$ ....(1)

Putting $(x^2 - 3x) = y$ in (1), we get $y^2 - 5y + 6 = 0$, which is a quadratic equation in y and can be rewritten as

$y^2 - 3y - 2y + 6 = 0$

$\Rightarrow \quad y(y - 3) - 2(y - 3) = 0$

$\Rightarrow \quad (y - 3)(y - 2) = 0$

Either $\quad y - 3 = 0$ or $y - 2 = 0$

$\Rightarrow \quad y = 3$ or $y = 2$.

Now, if $y = 3$, then $x^2 - 3x = 3 \quad [\because x^2 - 3x = y]$

$\Rightarrow \quad x^2 - 3x - 3 = 0$

$\Rightarrow \quad x = \frac{-(-3) \pm \sqrt{9 + 12}}{2} = \frac{3 \pm \sqrt{21}}{2}$

If $\quad y = 2$ then $x^2 - 3x = 2$

$\Rightarrow \quad x^2 - 3x - 2 = 0$

$\Rightarrow \quad x = \frac{-(-3) \pm \sqrt{9 + 8}}{2}$

$\Rightarrow \quad x = \frac{3 \pm \sqrt{17}}{2}$

Hence, the solutions of the given equation are $x = \frac{3 \pm \sqrt{17}}{2}$ and $\frac{3 \pm \sqrt{21}}{2}$.

VSA.10 Solve : $4^x - 3.2^{x+2} + 32 = 0$

Sol. We have,

$4^x - 3.2^{x+2} + 32 = 0$

$\Rightarrow \quad (2^2)^x - 3.2^x . 2^2 + 32 = 0$

$\Rightarrow \quad (2^x)^2 - 12(2^x) + 32 = 0 \quad$ ....(1)

Putting $2^x = y$ in (1), we get

$y^2 - 12y + 32 = 0$

$\Rightarrow \quad y^2 - 4y - 8y + 32 = 0$

$\Rightarrow \quad y(y - 4) - 8(y - 4) = 0$

$\Rightarrow \quad (y - 4)(y - 8) = 0$

Either $\quad y - 4 = 0$ or $y - 8 = 0$

$\Rightarrow \quad y = 4$ or $y = 8$

Now, if y = 4, then

$2^x = 4$

$\Rightarrow \quad 2^x = 2^2$

$\Rightarrow \quad x = 2$ $\qquad [\because 2^x = y]$

If y = 8, then

$2^x = 8$

$\Rightarrow \quad 2^x = 2^3$

$\Rightarrow \quad x = 3$

Hence, the required roots of given equation are 2, 3.

VSA.11 Solve the biquadratic i.e. the fourth degree equation $x^4 - 8x^2 - 9 = 0$.

Sol. Putting $x^2 = y$, the given equation reduces to

$y^2 - 8y - 9 = 0$

$\Leftrightarrow \quad y^2 - 9y + y - 9 = 0$

$\Leftrightarrow \quad (y - 9)(y + 1) = 0$

$\Leftrightarrow \quad y = 9 \text{ or } y = -1$

Now, $y = 9$

$\Rightarrow \quad x^2 = 9$

$\Rightarrow \quad x = \pm 3$

And, $y = -1$

$\Rightarrow \quad x^2 = -1$

$\Rightarrow \quad x = \pm\sqrt{-1} = \pm i.$

Hence, the roots of the given equation are –3, 3, i and –i.

VSA.12 Solve the equation : $x^2 + y^2 = 185$.

Sol. We have,

$x^2 + y^2 = 185 \qquad ....(1)$

and $\quad x + y = 19 \qquad ....(2)$

(2) implies

$y = 19 - x.$

Putting this value of y in (1), we get

$x^2 + (19 - x)^2 = 185$

i.e., $\quad x^2 + 316 + x^2 - 38x = 185$

$\Rightarrow \quad 2x^2 - 38x + 176 = 0$

$\Rightarrow \quad x^2 - 19x + 88 = 0$

$\Rightarrow \quad (x - 8)(x - 11) = 0$

$\Rightarrow \quad x = 8, 11$

Either $x = 8$ or $y = 11$.

$\therefore \quad y = 19 - x = 19 - 8 = 11$ $\qquad \therefore \quad y = 19 - x = 19 - 11 = 8$

$\therefore$ The solution is x = 8, y = 11; x = 11, y = 8.

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

SHORT ANSWER TYPE QUESTIONS :

SA.1 Solve : $2\left(x^2+\frac{1}{x^2}\right) - 9\left(x+\frac{1}{x}\right) + 14 = 0$

Sol. We have,

$$2\left(x^2+\frac{1}{x^2}\right) - 9\left(x+\frac{1}{x}\right) + 14 = 0 \quad ......(1)$$

$$\left[\begin{array}{ll}\text{Squaring} & x^2+\frac{1}{x^2}+2(x)\frac{1}{x}=y^2 \\ \Rightarrow & x^2+\frac{1}{x^2}+2=y^2\end{array}\right]$$

Putting $x + \frac{1}{x} = y$

$\Rightarrow$ $x^2 + \frac{1}{x^2} = y^2 - 2$ in (1), we get

$2(y^2 - 2) - 9(y) + 14 = 0$

$\Rightarrow$ $2y^2 - 4 - 9y + 14 = 0$

$\Rightarrow$ $2y^2 - 9y + 10 = 0$

$\Rightarrow$ $2y^2 - 5y - 4y + 10 = 0$

$\Rightarrow$ $y(2y - 5) - 2(2y - 5) = 0$

$\Rightarrow$ $(2y - 5)(y - 2) = 0$

Either $2y - 5 = 0$ or $y - 2 = 0$

$\Rightarrow$ $y = \frac{5}{2}$ or $y = 2$

Now, if $y = \frac{5}{2}$, then $x + \frac{1}{x} = \frac{5}{2}$ $\left[\because y = x+\frac{1}{x}\right]$

$\Rightarrow$ $\frac{x^2+1}{x} = \frac{5}{2}$

$\Rightarrow$ $2(x^2 + 1) = 5x$

$\Rightarrow$ $2x^2 - 5x + 2 = 0$

$\Rightarrow$ $x = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4} = 2$ or $\frac{1}{2}$

Now, if $y = 2$, then $x + \frac{1}{x} = 2$

$\Rightarrow$ $\frac{x^2+1}{x} = 2$

$\Rightarrow$ $x^2 - 2x + 1 = 0$

$\Rightarrow$ $(x - 1)^2 = 0$

$\Rightarrow$ $x = 1$

Hence, the solutions of the given equation are $x = 1$, $x = 2$, $x = \frac{1}{2}$.

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

SA.2 Solve the following equation : $\sqrt{x^2+4x-21} + \sqrt{x^2-x-6} = \sqrt{6x^2-5x-39}$

Sol. We have,

$$\sqrt{x^2+4x-21} + \sqrt{x^2-x-6} = \sqrt{6x^2-5x-39}$$

$\Rightarrow \quad \sqrt{(x+7)(x-3)} + \sqrt{(x-3)(x+2)} = \sqrt{(x-3)(6x+13)}$

$\Rightarrow \quad \sqrt{x-3}\,\{\sqrt{x+7} + \sqrt{x+2} - \sqrt{6x+13}\} = 0$

$\Rightarrow \quad \sqrt{x-3} = 0$ or $\sqrt{x+7} + \sqrt{x+2} - \sqrt{6x+13} = 0$

$\Rightarrow \quad x = 3$ or $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Now, $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

$\Rightarrow \quad (\sqrt{x+7} + \sqrt{x+2})^2 = 6x + 13$ [Squaring both sides]

$\Rightarrow \quad x + 7 + x + 2 + 2\sqrt{(x+7)(x+2)} = 6x + 13$

$\Rightarrow \quad 2x + 9 + 2\sqrt{(x+7)(x+2)} = 6x + 13$

$\Rightarrow \quad 2\sqrt{(x+7)(x+2)} = 4x + 4$

$\Rightarrow \quad \sqrt{(x+7)(x+2)} = 2(x + 1)$

$\Rightarrow \quad (x + 7)(x + 2) = 4(x + 1)^2$ [Squaring both sides]

$\Rightarrow \quad x^2 + 9x + 14 = 4(x^2 + 2x + 1)$

$\Rightarrow \quad 3x^2 - x - 10 = 0$

$\Rightarrow \quad (x - 2)(3x + 5) = 0$

$\Rightarrow \quad x = 2$ or $x = -\frac{5}{3}$.

But, $x = -\frac{5}{3}$ does not satisfy the given equation.

Hence, the roots of the given equation are 2 and 3.

SA.3 Solve : $\left(x+\frac{1}{x}\right)^2 - \frac{3}{2}\left(x-\frac{1}{x}\right) = 4;\ x \neq 0.$

Sol. We have,

$$\left(x+\frac{1}{x}\right)^2 - \frac{3}{2}\left(x-\frac{1}{x}\right) = 4 \qquad ......(1)$$

Putting $x - \frac{1}{x} = y$,

so that $\left(x+\frac{1}{x}\right)^2 = \left(x-\frac{1}{x}\right)^2 + 4 = y^2 + 4$ in (1), we get

$$(y^2 + 4) - \frac{3}{2}(y) = 4$$

⇒ $2y^2 - 3y = 0$

⇒ $y(2y - 3) = 0$

Either $y = 0$ or $2y - 3 = 0$ i.e. $y = \frac{3}{2}$

Now, if $y = 0$, then

$x - \frac{1}{x} = 0$ $\left[\because x - \frac{1}{x} = y\right]$

⇒ $x^2 - 1 = 0$

⇒ $x^2 = 1$

⇒ $x = \pm 1$.

If $y = \frac{3}{2}$, then

$x - \frac{1}{x} = \frac{3}{2}$

⇒ $\frac{x^2 - 1}{x} = \frac{3}{2}$

⇒ $2x^2 - 2 = 3x$

⇒ $2x^2 - 3x - 2 = 0$

⇒ $2x^2 - 4x + x - 2 = 0$

⇒ $2x(x - 2) + 1(x - 2) = 0$

⇒ $(x - 2)(2x + 1) = 0$

⇒ $x = 2$ or $x = -\frac{1}{2}$.

Hence, the roots (solutions) of the given equation are

$x = \pm 1,\ x = 2,\ x = -\frac{1}{2}$.

SA.4 Solve the equation : $\sqrt{5x^2 - 6x + 8} - \sqrt{5x^2 - 6x - 7} = 1$.

Sol. Let $5x^2 - 6x = y$. Then,

$\sqrt{5x^2 - 6x + 8} - \sqrt{5x^2 - 6x - 7} = 1$.

⇒ $\sqrt{y + 8} - \sqrt{y - 7} = 1$

⇒ $(\sqrt{y + 8} - \sqrt{y - 7})^2 = 1$

⇒ $y + 8 + y - 7 - 2\sqrt{y^2 + y - 56} = 1$

⇒ $2y + 1 = 2\sqrt{y^2 + y - 56} + 1$

⇒ $y = \sqrt{y^2 + y - 56}$

⇒ $y^2 = y^2 + y - 56$

$\Rightarrow \quad y = 56$

$\Rightarrow \quad 5x^2 - 6x = 56 \qquad [\because y = 5x^2 - 6x]$

$\Rightarrow \quad 5x^2 - 6x - 56 = 0$

$\Rightarrow \quad (5x + 14)(x - 4) = 0$

$\Rightarrow \quad x = 4, -\frac{14}{5}.$

Clearly, both the values satisfy the given equation.

Hence, the roots of the given equation are 4 and $-\frac{14}{5}$.

SA.5 Solve the equation $x^2 + \left(\frac{ax}{x+a}\right)^2 = 3a^2, x \neq -a.$

Sol. We have,

$$x^2 + \left(\frac{ax}{x+a}\right)^2 = 3a^2, x \neq -a.$$

$\Rightarrow \quad \left(x - \frac{ax}{x+a}\right)^2 + 2 \cdot x \cdot \frac{ax}{x+a} = 3a^2 \qquad$ (Using $A^2 + B^2 = (A - B)^2 + 2AB$)

$\Rightarrow \quad \left(\frac{x^2}{x+a}\right)^2 + 2a\left(\frac{x^2}{x+a}\right) = 3a^2$

Let $\quad y = \frac{x^2}{x+a}$

$\therefore \quad (1) \quad y^2 + 2ay - 3a^2 = 0$

$\Rightarrow \quad y = \frac{-2a \pm \sqrt{4a^2 + 12a^2}}{2} = \frac{-2a \pm 4a}{2} = a, -3a.$

Either $\quad y = a \qquad$ or $\quad y = -3a$

$\therefore \quad \frac{x^2}{x+a} = a \qquad \therefore \quad \frac{x^2}{x+a} = -3a$

or $\quad x^2 - ax - a^2 = 0 \qquad$ or $\quad x^2 + 3ax + 3a^2 = 0$

$\therefore \quad x = \frac{a \pm \sqrt{a^2 + 4a^2}}{2a} = \frac{a \pm \sqrt{5}a}{2} \qquad$ or $\quad x = \frac{-3a \pm \sqrt{9a^2 - 12a^2}}{2} = \frac{-3a \pm \sqrt{3}ai}{2}$

$\quad = \frac{a}{2}(1 \pm \sqrt{5}) \qquad = \frac{a}{2}(-3 \pm \sqrt{3}i)$

$\therefore$ The roots are $\frac{a}{2}(1 \pm \sqrt{5}), \frac{a}{2}(-3 \pm \sqrt{3}i).$

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

SA.6 Solve the equation : $8\sqrt{\frac{x}{x+3}} - \sqrt{\frac{x+3}{x}} = 2$

Sol. We have,

$$8\sqrt{\frac{x}{x+3}} - \sqrt{\frac{x+3}{x}} = 2$$

Let $y = \sqrt{\frac{x}{x+3}}$

$\therefore$ (1) $8y - \frac{1}{y} = 2$

$\Rightarrow$ $8y^2 - 1 = 2y$

$\Rightarrow$ $8y^2 - 2y - 1 = 0$

$\therefore$ $y = \frac{2 \pm \sqrt{4+32}}{16} = \frac{2 \pm 6}{16} = \frac{1}{2}, -\frac{1}{4}.$

Either $y = \frac{1}{2}$

$\Rightarrow$ $\sqrt{\frac{x}{x+3}} = \frac{1}{2}$

$\Rightarrow$ $\frac{x}{x+3} = \frac{1}{4}$

$\Rightarrow$ $4x = x + 3$

$\Rightarrow$ $x = \frac{3}{3} = 1$

or $y = -\frac{1}{4}$

$\Rightarrow$ $\sqrt{\frac{x}{x+3}} = -\frac{1}{4}$

This is impossible, because L.H.S. is non-negative.

$\therefore$ The root is 1.

SA.7 Solve the equation : $(15 + 4\sqrt{14})^{x^2 - 2x} + (15 - 4\sqrt{14})^{x^2 - 2x} = 30.$

Sol. We have,

$(15 + 4\sqrt{14})^{x^2 - 2x} + (15 - 4\sqrt{14})^{x^2 - 2x} = 30$ ......(1)

Now $(15 + 4\sqrt{14}) + (15 - 4\sqrt{14}) = 225 - 224 = 1$

$\therefore$ $15 - 4\sqrt{14} = \frac{1}{15 + 4\sqrt{14}}$

$\therefore$ (1) $(15 + 4\sqrt{14})^{x^2 - 2x} + \frac{1}{(15+4\sqrt{14})^{x^2-2x}} = 30$ ......(2)

Let $y = (15 + 4\sqrt{14})^{x^2 - 2x}$

$\therefore$ (2) $y + \frac{1}{y} = 30$

$\Rightarrow$ $y^2 + 1 = 30y$

$\Rightarrow$ $y^2 - 30y + 1 = 0$

$\therefore$ $y = \frac{30 \pm \sqrt{900-4}}{2} = \frac{30 \pm \sqrt{896}}{2} = 15 \pm 4\sqrt{14}$

Either $y = 15 \pm 4\sqrt{14}$

$\Rightarrow$ $(15 \pm 4\sqrt{14})^{x^2 - 2x} = 15 + 4\sqrt{14}$

$\Rightarrow$ $x^2 - 2x = 1$

$\Rightarrow$ $x^2 - 2x - 1 = 0$

$\therefore$ $x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

or $y = 15 - 4\sqrt{14}$

$\Rightarrow$ $(15 + 4\sqrt{14})^{x^2 - 2x} = 15 - 4\sqrt{14}$

$\Rightarrow$ $(15 + 4\sqrt{14})^{x^2 - 2x} = (15 + 4\sqrt{14})^{-1}$

$\Rightarrow$ $x^2 - 2x = -1$

$\Rightarrow$ $x^2 - 2x + 1 = 0$

$\Rightarrow$ $(x - 1)^2 = 0$ or $x = 1, 1$.

$\therefore$ The roots are $1 \pm \sqrt{2}$, 1, 1.

SA.8 Solve : $\left(\frac{2x+3}{x+1}\right) + 6\left(\frac{x+1}{2x+3}\right) - 7 = 0$.

Sol. We have,

$$\left(\frac{2x+3}{x+1}\right) + 6\left(\frac{x+1}{2x+3}\right) - 7 = 0$$

Putting $\frac{2x+3}{x+1} = y$ in (1), we get

$$y + \frac{6}{y} - 7 = 0$$

$\Rightarrow$ $y^2 + 6 - 7y = 0$ [Multiplying by y]

$\Rightarrow$ $y^2 - 7y + 6 = 0$

$\Rightarrow$ $y^2 - 6y - y + 6 = 0$

$\Rightarrow$ $y(y - 6) - 1(y - 6) = 0$

$\Rightarrow$ $(y - 6)(y - 1) = 0$

Either $y - 6 = 0$ or $y - 1 = 0$

$\Rightarrow$ $y = 6$ or $y = 1$

Now, if $y = 6$, then

$\frac{2x+3}{x+1} = 6$ $\left[\because \frac{2x+3}{x+1} = y\right]$

$\Rightarrow$ $2x + 3 = 6x + 6$

$\Rightarrow$ $4x = -3$

$\Rightarrow$ $x = -\frac{3}{4}$

Now, if $y = 1$, then

$\frac{2x+3}{x+1} = 1$

$\Rightarrow$ $2x + 3 = x + 1$

$\Rightarrow$ $x = -2$

Hence, the required roots of given equation are $-2. -\frac{3}{4}$.

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

SA.9 Solve : $x^2 + \frac{x^2}{(x+1)^2} = 3, x \neq -1.$

Sol. We have,

$$x^2 + \frac{x^2}{(x+1)^2} = 3$$

$$\Leftrightarrow \quad x^2 + \left(\frac{x}{x+1}\right)^2 = 3$$

$$\Leftrightarrow \quad \left(x - \frac{x}{x+1}\right)^2 + 2 . x . \left(\frac{x}{x+1}\right) = 3$$

[using $a^2 + b^2 = (a - b)^2 + 2ab$]

$$\Leftrightarrow \quad \left(\frac{x^2 + x - x}{x+1}\right)^2 + 2\left(\frac{x^2}{x+1}\right) = 3$$

$$\Leftrightarrow \quad \left(\frac{x^2}{x+1}\right)^2 + 2\left(\frac{x^2}{x+1}\right) - 3 = 0$$

$$\Leftrightarrow \quad y^2 + 2y - 3 = 0, \text{ where } y = \frac{x^2}{x+1}.$$

$$\Leftrightarrow \quad (y + 3)(y - 1) = 0$$

$$\Leftrightarrow \quad y = 1 \text{ or } y = -3.$$

Now, $y = 1$

$$\Rightarrow \quad \frac{x^2}{x+1} = 1$$

$$\Leftrightarrow \quad x^2 - x - 1 = 0$$

$$\Rightarrow \quad x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

And, $y = -3$

$$\Rightarrow \quad \frac{x^2}{x+1} = -3$$

$$\Leftrightarrow \quad x^2 + 3x + 3 = 0$$

$$\Rightarrow \quad x = \frac{-3 \pm \sqrt{9-12}}{2} = \frac{-3 \pm i\sqrt{3}}{2}$$

Hence, the roots of the given equation $\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}, \frac{-3+i\sqrt{3}}{2}$ and $\frac{-3-i\sqrt{3}}{2}$.

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

SA.10 Solve : $2^{2x+8} + 1 = 32.2^x$.

Sol. The given equation is

$$2^{2x+8} + 1 = 32.2^x$$

$\Rightarrow$ $2^{2x} . 2^8 - 32.2^x + 1 = 0$

$\Rightarrow$ $256.2^{2x} - 32.2^x + 1 = 0$ [$\because 2^8 = 256$]

$\Rightarrow$ $256(2^x)^2 - 32.2^x + 1 = 0$.

Substituting $2^x = t$, we get

$$256t^2 - 32t + 1 = 0,$$

$\therefore$ $t = \dfrac{-(-32) \pm \sqrt{(-32)^2 - 4.256.1}}{2.256}$ [$\because$ a = 256, b, = −32, c = 1]

$= \dfrac{32 \pm 0}{512} = \dfrac{1}{16}, \dfrac{1}{16}$.

But, $t = 2^x$, therefore, $2^x = \dfrac{1}{16}, \dfrac{1}{16}$.

$\Rightarrow$ $2^x = \dfrac{1}{2^4}, \dfrac{1}{2^4} = 2^{-4}, 2^{-4}$

$\Rightarrow$ $x = -4, -4$

$\therefore$ The roots of the given equation are −4, −4.

SA.11 Solve the equation : $(x^2 + x - 6)(x^2 - 3x - 4) = 24$.

Sol. The given equation is

$$(x^2 + x - 6)(x^2 - 3x - 4) = 24$$

$\Rightarrow$ $(x + 3)(x - 2)(x - 4)(x + 1) = 24$

$\Rightarrow$ $[(x + 3)(x - 4)][(x - 2)(x + 1)] - 24 = 0$ [$\because 3 + (-4) = -1$ & $(-2) + 1 = -1$]

$\Rightarrow$ $[x^2 - x - 12][x^2 - x - 2] - 24 = 0$

$\Rightarrow$ $(t - 12)(t - 2) - 24 = 0$, where $t = x^2 - x$

$\Rightarrow$ $t^2 - 12t - 2t + 24 - 24 = 0$

$\Rightarrow$ $t^2 - 14t = 0$

$\Rightarrow$ $t(t - 14) = 0$

$\Rightarrow$ $t = 0$ or $t - 14 = 0$

$\Rightarrow$ $t = 0, 14$.

But $t = x^2 - x$, therefore,

Either $x^2 - x = 0$ or $x^2 - x = 14$

$\Rightarrow$ $x(x - 1) = 0$ $\Rightarrow$ $x^2 - x - 14 = 0$

$\Rightarrow$ $x = 0$ or $x - 1 = 0$ $\Rightarrow$ $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4.1.(-14)}}{2.1}$

$\Rightarrow$ $x = 0$, $x = 1$ $\Rightarrow$ $x = \dfrac{1 \pm \sqrt{57}}{2}$

$\therefore$ 1, 0, $\dfrac{1 \pm \sqrt{57}}{2}$ are the roots of the given equation.

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

SA.12 Solve the equation : $\frac{1}{\sqrt{1-x}+1} + \frac{1}{\sqrt{1+x}-1} = \frac{1}{x}$.

Sol. We have,

$$\frac{1}{\sqrt{1-x}+1} + \frac{1}{\sqrt{1+x}-1} = \frac{1}{x}$$

Rationalising each term on the L.H.S., we get

$$\left(\frac{1}{\sqrt{1-x}+1}\times\frac{\sqrt{1-x}-1}{\sqrt{1-x}-1}\right) + \left(\frac{1}{\sqrt{1+x}-1}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}\right) = \frac{1}{x}$$

$$\Rightarrow \quad \frac{\sqrt{1-x}-1}{1-x-1} + \frac{\sqrt{1+x}+1}{1+x-1} = \frac{1}{x}$$

$$\Rightarrow \quad \frac{\sqrt{1-x}-1}{-x} + \frac{\sqrt{1+x}+1}{x} = \frac{1}{x}$$

Multiplying by x, we get

$$(1 - \sqrt{1-x}) + (\sqrt{1+x} + 1) = 1$$

$$\Rightarrow \quad \sqrt{1+x} - \sqrt{1-x} = -1 \quad \text{........(1)}$$

Squaring, we get

$$(1 + x) + (1 - x) - 2\sqrt{1+x}\,\sqrt{1-x} = 1 \quad \text{or} \quad 2\sqrt{1-x^2} = 1$$

Squaring again, we get

$$4(1 - x^2) = 1 \quad \text{or} \quad x^2 = \frac{3}{4} \quad \text{i.e., } x = \pm\sqrt{\frac{3}{2}}$$

It may be checked that $x = -\sqrt{\frac{3}{2}}$ satisfies (1) and $x = \sqrt{\frac{3}{2}}$ does not satisfy it.

$\therefore$ The only root is $-\sqrt{\frac{3}{2}}$.

SA.13 Solve : $(x + 1)(x + 2)(x + 3)(x + 4) + 1 = 0$

Sol. We have,

$$(x + 1)(x + 2)(x + 3)(x + 4) + 1 = 0$$

This can be written as

$$[(x + 1)(x + 4)][(x + 2)(x + 3)] + 1 = 0 \quad [\because 1 + 4 = 2 + 3 = 5]$$

$$\Rightarrow \quad (x^2 + 5x + 4)(x^2 + 5x + 6) + 1 = 0 \quad \text{........(1)}$$

Putting $x^2 + 5x = y$ in (1), we get

$$(y + 4)(y + 6) + 1 = 0$$

$$\Rightarrow \quad y^2 + 10y + 24 + 1 = 0$$

$$\Rightarrow \quad y^2 + 10y + 25 = 0$$

$$\Rightarrow \quad (y + 5)^2 = 0$$

$$\Rightarrow \quad y = -5, -5$$

Now, if y = –5, then

$x^2 + 5x = -5$ $\quad [\because x^2 + 5x = y]$

$\Rightarrow \quad x^2 + 5x + 5 = 0$

$$\Rightarrow \quad x = \frac{-5 \pm \sqrt{25-20}}{2}$$

$$\Rightarrow \quad x = \frac{-5 \pm \sqrt{5}}{2}$$

Hence, the roots of the given equation are $\frac{-5 \pm \sqrt{5}}{2}$ (repeated).

SA.14 Solve the equation : $\sqrt{x^2 - ax + b} + \sqrt{x^2 - ax + c} = \sqrt{b} + \sqrt{c}$

Sol. We have,

$$\sqrt{x^2 - ax + b} + \sqrt{x^2 - ax + c} = \sqrt{b} + \sqrt{c} \quad \text{........(1)}$$

Let A = $\sqrt{x^2 - ax + b}$ and B = $\sqrt{x^2 - ax + c}$

$\therefore$ (1) $\quad A + B = \sqrt{b} + \sqrt{c}$ $\quad$ ....(2)

Now $\quad A^2 + B^2 = (x^2 - ax + b) - (x^2 - ax + c) = b - c$

$\therefore \quad A^2 + B^2 = b - c$ $\quad$ ....(3)

Dividing (3) by (2), we get

$$\frac{A^2 + B^2}{A + B} = \frac{b - c}{\sqrt{b} + \sqrt{c}}$$

$\Rightarrow \quad A - B = \sqrt{b} - \sqrt{c}$ $\quad$ ....(4)

(2) + (4)

$\Rightarrow \quad 2A = 2\sqrt{b} \quad$ i.e., $A = \sqrt{b}$

$\therefore \quad \sqrt{x^2 - ax + b} = \sqrt{b}$ or $x^2 - ax + b = b$

$\Rightarrow \quad x(x - a) = 0$

$\therefore \quad x = 0, a$

$\therefore$ The roots are 0, a.

SA.15 Solve the equation : $\frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}} = \frac{a}{x}$.

Sol. We have,

$$\frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}} = \frac{a}{x}$$

Applying componendo and dividendo, we get

$$\frac{(\sqrt{a+x} + \sqrt{a-x}) + (\sqrt{a+x} - \sqrt{a-x})}{(\sqrt{a+x} + \sqrt{a-x}) - (\sqrt{a+x} - \sqrt{a-x})} = \frac{a+x}{a-x}$$

$\Rightarrow \quad \dfrac{2\sqrt{a+x}}{2\sqrt{a-x}} = \dfrac{a+x}{a-x}$

$\Rightarrow \quad \sqrt{a+x}\,(a - x) = \sqrt{a-x}\,(a + x).$

$\Rightarrow \quad \sqrt{a+x}\,\sqrt{a-x}\ [\sqrt{a-x} - \sqrt{a+x}\,] = 0$

Either $\sqrt{a+x} = 0$ ....(1)

or $\sqrt{a-x} = 0$ ....(2)

or $\sqrt{a-x} - \sqrt{a+x} = 0$ ....(3)

(1) $\Rightarrow$ $a + x = 0$ i.e., $x = -a$

(2) $\Rightarrow$ $a - x = 0$ i.e., $x = a$

(3) $\Rightarrow$ $\sqrt{a-x} = \sqrt{a+x}$ $\Rightarrow$ $a - x = a + x$ $\Rightarrow$ $2x = 0$ i.e., $x = 0$

$x = 0$ does not satisfy the given equation, $\therefore$ The roots are $\pm a$.

SA.16 Solve : $(1 + x)^{2/3} + (1 - x)^{2/3} = 3(1 - x^2)^{1/3}$.

Sol. We have,

$(1 + x)^{2/3} + (1 - x)^{2/3} = 3(1 - x^2)^{1/3}$ ....(1)

Cubing both sides of (1), we get

$[(1 + x)^{2/3} + (1 - x)^{2/3}]^3 = [3(1 - x^2)^{1/3}]^3$

$\Rightarrow \quad (1 + x)^2 + (1 - x)^2 + 3(1 + x)^{2/3}(1 - x)^{2/3}[(1 + x)^{2/3} + (1 - x)^{2/3}] = 27(1 - x^2)$

$[\because (a + b)^3 = a^3 + b^3 + 3ab(a + b)]$

$\Rightarrow \quad (1 + x^2 + 2x) + (1 + x^2 - 2x) + 3(1 - x^2)^{2/3} . 3(1 - x^2)^{1/3} = 27 - 27x^2$ [using (1)]

$\Rightarrow \quad 2x^2 + 2 + 9(1 - x^2) = 27 - 27x^2$

$\Rightarrow \quad 29x^2 - 25 + 9(1 - x^2) = 0$

$\Rightarrow \quad 29x^2 - 25 + 9 - 9x^2 = 0$

$\Rightarrow \quad 20x^2 = 16$

$\Rightarrow \quad x^2 = \dfrac{16}{20} = \dfrac{4}{5}.$

$\therefore \quad x = \pm\dfrac{2}{\sqrt{5}}$

Hence, the required roots are $\dfrac{2}{\sqrt{5}}$ and $-\dfrac{2}{\sqrt{5}}$.

SA.17 Solve the system of equations : $(x + y)^2 - 2(x + y) = 15$, $xy = 6$.

Sol. We have,

$(x + y)^2 - 2(x + y) = 15$ ....(1)

and, $xy = 6$ ....(2)

Putting $x + y = u$ in (1) it reduces to

$u^2 - 2u - 15 = 0$

$\Rightarrow \quad (u - 5)(u + 3) = 0$

$\Rightarrow \quad u = 5$ or, $u = -3$

$\Rightarrow \quad x + y = 5$ or, $x + y = -3$

Now, $x + y = 5$ and $xy = 6$

$\Rightarrow$ $x(5 - x) = 6$ $[\because x + y = 5 \Rightarrow y = 5 - x]$

$\Rightarrow$ $x^2 - 5x + 6 = 0$

$\Rightarrow$ $(x - 2)(x - 3) = 0$

$\Rightarrow$ $x = 2$ or $x = 3$

Putting $x = 2$ and $x = 3$ respectively in $x + y = 5$, we get $y = 3$ and $y = 2$ respectively. Thus $x = 2, y = 3$ and $x = 3, y = 2$ are two solutions of the given system of equations.

Again, $x + y = -3$ and $xy = 6$

$\Rightarrow$ $x(-3 - x) = 6$

$\Rightarrow$ $x^2 + 3x + 6 = 0$

$$\Rightarrow \quad x = \frac{-3 \pm \sqrt{9-24}}{2} = \frac{-3 \pm i\sqrt{15}}{2}$$

Now, $x = \frac{-3 \pm i\sqrt{15}}{2}$ and $x + y = -3$

$$\Rightarrow \quad y = \frac{-3 \mp i\sqrt{15}}{2}$$

Thus, $x = \frac{-3 + i\sqrt{15}}{2}$, $y = \frac{-3 - i\sqrt{15}}{2}$

and, $x = \frac{-3 - i\sqrt{15}}{2}$, $y = \frac{-3 + i\sqrt{15}}{2}$

are two more solutions of the given system of equations.

Hence, the required roots are :

$$x = 2, y = 3; x = 3, y = 2; x = \frac{-3 + i\sqrt{15}}{2}, y = \frac{-3 - i\sqrt{15}}{2}$$

$$x = \frac{-3 - i\sqrt{15}}{2}, y = \frac{-3 + i\sqrt{15}}{2}.$$

SA.18 Solve the simultaneous equations : $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{5}{2}$; $x + y = 10$.

Sol. We have,

$$\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{5}{2} \qquad \text{........(1)}$$

and $x + y = 10$ ....(2)

Now, $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{5}{2}$

$$\Rightarrow \quad \frac{x + y}{\sqrt{xy}} = \frac{5}{2}$$

$\Rightarrow \quad \frac{10}{\sqrt{xy}} = \frac{5}{2}$ [using (2)]

$\Rightarrow \quad \sqrt{xy} = 4$

$\Rightarrow \quad xy = 16$

Thus, the given system of simultaneous equations reduces to

$x + y = 10$ and $xy = 16$

$\Rightarrow \quad y = 10 - x$ and $xy = 16$

$\Rightarrow \quad x(10 - x) = 16$

$\Rightarrow \quad x^2 - 10x + 16$

$\Rightarrow \quad (x - 2)(x - 8) = 0$

$\Rightarrow \quad x = 2$ or $x = 8$

Now $\quad x = 2$ and $x + y = 10 \Rightarrow y = 8$.

$x = 8$ and $x + y = 10 \Rightarrow y = 2$.

Hence, the required roots are $x = 2, y = 8$ and $x = 8, y = 2$.

SA.19 Solve the system of equations : $x^4 + y^4 = 82$; $x - y = 2$.

Sol. The given system of equation is

$x^4 + y^4 = 82$ ......(1)

$x - y = 2$ ......(2)

Let $x = u + v$ and $y = u - v$. Then,

$x - y = 2$

$\Rightarrow \quad 2v = 2$

$\Rightarrow \quad v = 1$.

Now, $\quad x^4 + y^4 = 82$

$\Rightarrow \quad (u + v)^4 + (u - v)^4 = 82$

$\Rightarrow \quad (u + 1)^4 + (u - 1)^4 = 82$

$\Rightarrow \quad (u^4 + 4u^3 + 6u^2 + 4u + 1) + (u^4 - 4u^3 + 6u^2 - 4u + 1) = 82$

$\Rightarrow \quad 2(u^4 + 6u^2 + 1) = 82$

$\Rightarrow \quad u^4 + 6u^2 + 1 = 41$

$\Rightarrow \quad u^4 + 6u^2 - 40 = 0$

$\Rightarrow \quad (u^2 + 10)(u^2 - 4) = 0$

$\Rightarrow \quad u^2 = 4$ or $u^2 = -10$.

$\Rightarrow \quad u = \pm 2$ or $u^2 = \pm i\sqrt{10}$

Thus, we have

$u = \pm 2, \pm i\sqrt{10}$ and $v = 1$.

$\therefore \quad x = u + v$

$\Rightarrow \quad x = 3, -1, 1 + i\sqrt{10}, 1 - i\sqrt{10}$

and $\quad y = u - v$

$\Rightarrow \quad y = 1, -3 - 1 + i\sqrt{10}, -1 - i\sqrt{10}$

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

SA.20 Solve the equation : $x^4 + y^4 = 257;\ x + y = 5$

Sol. We have,

$x^4 + y^4 = 257$ ......(1)

and $x + y = 5$ ......(2)

Let $x = u + v$

and $y = u - v$

Then, $x + y = u + v + u - v$

$\Rightarrow$ $x + y = 2u$ ......(3)

From (2) and (3), we get

$2u = 5$

$\Rightarrow$ $u = \frac{5}{2}$

Substituting $x = u + v$ and $y = u - v$ in (1), we get

$(u + v)^4 + (u - v)^4 = 257$

$\Rightarrow$ $(u^4 + 4u^3v + 6u^2v^2 + 4uv^3 + v^4) + (u^4 - 4u^3v + 6u^2v^2 - 4uv^3 + v^4) = 257$

$\Rightarrow$ $2(u^4 + 6u^2v^2 + v^4) = 257$

$\Rightarrow$ $2\left(\left(\frac{5}{2}\right)^4 + 6\left(\frac{5}{2}\right)^2 v^2 + v^4\right) = 257$ $\left[\because u = \frac{5}{2}\right]$

$\Rightarrow$ $16v^4 + 600v^2 - 1431 = 0$

$\Rightarrow$ $16v^4 - 36v^2 + 636v^2 - 1431 = 0$

$\Rightarrow$ $4v^2(4v^2 - 9) + 159(4v^2 - 9) = 0$

$\Rightarrow$ $(4v^2 - 9)(4v^2 + 159) = 0$

$\Rightarrow$ $v = \pm\frac{3}{2}$ or $v = \pm\frac{1}{2}\sqrt{159}\,i$

Now,

When $v = \frac{3}{2}$, $x = u + v = \frac{5}{2} + \frac{3}{2} = 4$ and $u - v = \frac{5}{2} - \frac{3}{2} = 1$

When $v = -\frac{3}{2}$, $x = u + v = \frac{5}{2} - \frac{3}{2} = 1$ and $u - v = \frac{5}{2} + \frac{3}{2} = 4$

When $v = \frac{1}{2}\sqrt{159}\,i$, $x = u + v = \frac{5}{2} + \frac{1}{2}\sqrt{159}\,i = \frac{5 + \sqrt{159}}{2}i$

and $y = u - v = \frac{5}{2} - \frac{1}{2}\sqrt{159}\,i = \frac{5 - \sqrt{159}}{2}i$

Similarly,

when $v = -\frac{1}{2}\sqrt{159}\,i$, $x = u + v = \frac{5 - \sqrt{159}}{2}i$, $y = u - v = \frac{5 + \sqrt{159}}{2}i$

Hence, $x = 4, 1, \frac{5 \pm \sqrt{159}i}{2}$ and $y = 1, 4, \frac{5 \mp \sqrt{159}i}{2}$.

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

SA.21 Solve the following equation : $(x + 2)(x + 3)(x + 8)(x + 12) = 4x^2$

Sol. We have,

$$3 \times 8 = 2 \times 12$$

So, we write the given equation in the following form :

$$\{(x + 2)(x + 12)\}\{(x + 3)(x + 8)\} = 4x^2$$

$\Rightarrow$ $(x^2 + 14x + 24)(x^2 + 11x + 24) = 4x^2$

Dividing throughout by x^2, we get

$$\left(x + 14 + \frac{24}{x}\right)\left(x + 11 + \frac{24}{x}\right) = 4$$

$\Rightarrow$ $(y + 14)(y + 11) = 4$, where $x + \frac{24}{x} = y$

$\Rightarrow$ $y^2 + 25y + 154 = 4$

$\Rightarrow$ $y^2 + 25y + 150 = 0$

$\Rightarrow$ $(y + 15)(y + 10) = 0$

$\Rightarrow$ $y = -15, -10$

If $y = -15$, then

$$x + \frac{24}{x} = -15$$

$\Rightarrow$ $x^2 + 15x + 24 = 0$

$\Rightarrow$ $x = \frac{-15 \pm \sqrt{29}}{2}$

If $y = -10$, then

$$x + \frac{24}{x} = -10$$

$\Rightarrow$ $x^2 + 10x + 24 = 0$

$\Rightarrow$ $(x + 4)(x + 6) = 0$

$\Rightarrow$ $x = -4, -6.$

Hence the roots of the given equation are $-4, -6, \frac{-15 \pm \sqrt{29}}{2}$.

SA.22 Solve the equation : $\frac{x-2}{x+2} + \frac{x+2}{x-2} = \frac{2(x+3)}{x-3}$

Sol. We have,

$$\frac{x-2}{x+2} + \frac{x+2}{x-2} = \frac{2(x+3)}{x-3}$$

$\Rightarrow$ $\frac{(x+2)-4}{x+2} + \frac{(x-2)+4}{x-2} = \frac{2(x+3)+12}{x-3}$

$\Rightarrow \quad 1 - \frac{4}{x+2} + 1 + \frac{4}{x-2} = 2 + \frac{12}{x-3}$

$\Rightarrow \quad -\frac{4}{x+2} + \frac{4}{x-2} = \frac{12}{x-3}$

$\Rightarrow \quad \frac{1}{x-2} - \frac{1}{x+2} = \frac{3}{x-3}$

$\Rightarrow \quad \frac{4}{x^2-4} = \frac{3}{x-3}$

$\Rightarrow \quad 4x - 12 = 3x^2 - 12$

$\Rightarrow \quad 3x^2 - 4x = 0$

$\Rightarrow \quad x(3x - 4) = 0$

$\Rightarrow \quad x = 0, \frac{4}{3}$

Hence, the roots of the given equation are 0, $\frac{4}{3}$.

SA.23 Solve : $\frac{x}{y} + \frac{y}{x} = 2\frac{1}{12}$; $x + y = 7$.

Sol. The given equations are :

$\frac{x}{y} + \frac{y}{x} = \frac{25}{12}$

i.e. $12x^2 - 25xy + 12y^2 = 0$ ....(1)

and $x + y = 7$ ....(2)

From (2), $y = 7 - x$ ....(3)

Putting the value of y from (3) in (1), we get :

$12x^2 - 25x(7 - x) + 12(7 - x)^2 = 0$

$\Rightarrow \quad 12x^2 - 175x + 25x^2 + 588 - 168x + 12x^2 = 0$

$\Rightarrow \quad 49x^2 - 343x + 588 = 0$

$\Rightarrow \quad x^2 - 7x + 12 = 0$

$\Rightarrow \quad x^2 - 3x - 4x + 12 = 0$

$\Rightarrow \quad x(x - 3) - 4(x - 3) = 0$

$\Rightarrow \quad (x - 3)(x - 4) = 0$

Either $x - 3 = 0$ or $x - 4 = 0$

i.e. $x = 3$ or $x = 4$.

(a) When $x = 3$, then from (3), $y = 7 - 3 = 4$.

(b) When $x = 4$, then from (3), $y = 7 - 4 = 3$.

Hence the solutions are $x = 3$, $y = 4$ and $x = 4$, $y = 3$.

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

LONG ANSWER TYPE QUESTIONS :

LA.1 Solve the equation : $12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0$.

Sol. The given equation is

$$12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0 \qquad ....(1)$$

In the equation the coefficients of the terms equidistant from the two ends are equal.
So, first divide both sides by x^2 and then regroup the terms containing same coefficient.
Dividing both sides of (1) by x^2, we get

$$12x^2 - 56x + 89 - \frac{56}{x} + \frac{12}{x^2} = 0$$

$$\Rightarrow \quad 12\left(x^2 + \frac{1}{x^2}\right) - 56\left(x + \frac{1}{x}\right) + 89 = 0$$

$$\Rightarrow \quad 12\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 56\left(x + \frac{1}{x}\right) + 89 = 0$$

$$\Rightarrow \quad 12\left(x + \frac{1}{x}\right)^2 - 56\left(x + \frac{1}{x}\right) + 65 = 0$$

$$\Rightarrow \quad 12y^2 - 56y + 65 = 0, \text{ where } y = x + \frac{1}{x}.$$

$$\Rightarrow \quad 12y^2 - 26y - 30y + 65 = 0$$

$$\Rightarrow \quad (6y - 13)(2y - 5) = 0$$

$$\Rightarrow \quad y = \frac{13}{6} \text{ or } y = \frac{5}{2}.$$

If $y = \frac{13}{6}$, then

$$x + \frac{1}{x} = \frac{13}{6}$$

$$\Rightarrow \quad 6x^2 - 13x + 6 = 0$$

$$\Rightarrow \quad (3x - 2)(2x - 3) = 0$$

$$\Rightarrow \quad x = \frac{2}{3}, \frac{3}{2}.$$

If $y = \frac{5}{6}$, then

$$x + \frac{1}{x} = \frac{5}{2}$$

$$\Rightarrow \quad 2x^2 - 5x + 2 = 0$$

$$\Rightarrow \quad (x - 2)(2x - 1) = 0$$

$$\Rightarrow \quad x = 2, \frac{1}{2}.$$

Hence, the roots of the given equation are 2, $\frac{1}{2}, \frac{2}{3}, \frac{3}{2}$.

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

LA.2 Solve : (x + 1) (2x + 3) (2x + 5) (x + 3) = 945.

Sol. We have,

$$(x + 1)(2x + 3)(2x + 5)(x + 3) = 945$$

$$\Rightarrow \quad (x + 1)\left[2\left(x+\frac{3}{2}\right)\right]\left[2\left(x+\frac{5}{2}\right)\right](x + 3) = 945$$

$$\Rightarrow \quad 4[(x + 1)(x + 3)]\left[\left(x+\frac{3}{2}\right)\left(x+\frac{5}{2}\right)\right] = 945$$

$$\Rightarrow \quad 4(x^2 + 4x + 3)\left(x^2+4x+\frac{15}{4}\right) = 945 \qquad ....(1)$$

Putting $x^2 + 4x = y$ in (1), we get

$$\Rightarrow \quad 4(y + 3)\left(y+\frac{15}{4}\right) = 945$$

$$\Rightarrow \quad 4(y + 3)\,\frac{4y+15}{4} = 945$$

$$\Rightarrow \quad 4y^2 + 12y + 15y + 45 - 945 = 0$$

$$\Rightarrow \quad 4y^2 + 27y - 900 = 0$$

$$\Rightarrow \quad y = \frac{-27 \pm \sqrt{(27)^2 - 4.4(-900)}}{2.4} = \frac{-27 \pm \sqrt{15129}}{8}$$

$$= \frac{-27 \pm 123}{8} = 12, -\frac{75}{4}$$

Now, if y = 12, then

$$x^2 + 4x = 12 \qquad [\because x^2 + 4x = y]$$

$$\Rightarrow \quad x^2 + 4x - 12 = 0$$

$$\Rightarrow \quad (x + 6)(x - 2) = 0$$

$$\Rightarrow \quad x = -6, 2$$

If $y = -\frac{75}{4}$, then

$$x^2 + 4x = -\frac{75}{4}$$

$$\Rightarrow \quad 4x^2 + 16x + 75 = 0$$

$$\Rightarrow \quad x = \frac{-16 \pm \sqrt{256 - 1200}}{8} = \frac{-16 \pm \sqrt{-59 \times 16}}{8} = \frac{-4 \pm i\sqrt{59}}{2}$$

Hence, the roots of the given equation are $-6, 2, \frac{-4 \pm i\sqrt{59}}{2}$.

Problem Based on Different Form of Equations (Reducible to Quadratic Equations)

LA.3 Solve the system of equations : $\left(3-\frac{6y}{x+y}\right)^2 + \left(3+\frac{6y}{x-y}\right)^2 = 82$; $3x + 7y = 26$.

Sol. We have,

$$\left(3-\frac{6y}{x+y}\right)^2 + \left(3+\frac{6y}{x-y}\right)^2 = 82$$

$$\Rightarrow \left(\frac{3x-3y}{x+y}\right)^2 + \left(\frac{3x+3y}{x-y}\right)^2 = 82$$

$$\Rightarrow 9\left(\frac{x-y}{x+y}\right)^2 + 9\left(\frac{x+y}{x-y}\right)^2 = 82$$

Let $\frac{x+y}{x-y} = u$. Then, the above equation becomes

$$9u^2 + \frac{9}{u^2} = 82$$

$\Rightarrow$ $9u^4 - 82u^2 + 9 = 0$

$\Rightarrow$ $9u^4 - 81u^2 - u^2 + 9 = 0$

$\Rightarrow$ $(9u^2 - 1)(u^2 - 9) = 0$

$\Rightarrow$ $u = \pm 3$ or $u = \pm\frac{1}{3}$.

Case I : When u = 3

In this case, u = 3

$\Rightarrow$ $\frac{x+y}{x-y} = 3$

$\Rightarrow$ $x + y = 3x - 3y$

$\Rightarrow$ $x = 2y$

Now, $x = 2y$ and $3x + 7y = 26$

$\Rightarrow$ $x = 4$, $y = 2$.

Case II : When u = –3

In this case, u = –3

$\Rightarrow$ $\frac{x+y}{x-y} = -3$

$\Rightarrow$ $x + y = -3x + 3y$

$\Rightarrow$ $y = 2x$

Now, $y = 2x$ and $3x + 7y = 26$

$\Rightarrow$ $x = \frac{26}{17}$, $y = \frac{52}{17}$.

Case III : When $u = \frac{1}{3}$

In this case, $u = \frac{1}{3}$

$\Rightarrow \quad \frac{x+y}{x-y} = \frac{1}{3}$

$\Rightarrow \quad 3x + 3y = x - y$

$\Rightarrow \quad x = -2y$

Now, $\quad x = -2y$ and $3x + 7y = 26$

$\Rightarrow \quad x = -52,\ y = 26.$

Case IV : When $u = -\frac{1}{3}$.

In this case, $u = -\frac{1}{3}$.

$\Rightarrow \quad \frac{x+y}{x-y} = -\frac{1}{3}$

$\Rightarrow \quad 3x + 3y = -x + y$

$\Rightarrow \quad 2x = -y.$

Now, $\quad 2x = -y$ and $3x + 7y = 26$

$\Rightarrow \quad x = -\frac{26}{11},\ y = \frac{52}{11}.$