# QUADRATIC EQUATION

## POLYNOMIAL

An algebraic expression of the form $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots\ldots\ldots a_{n-1}x^{n-1} + a_nx^n$ where $a_0, a_1, a_2 \ldots\ldots\ldots a_n$ are constants ($a_n \neq 0$) and n is a non negative integer is called n degree polynomial in x.

**1.1 Real Polynomial :** Let $a_0, a_1, a_2 \ldots\ldots\ldots\ldots a_n$ an be real numbers and x is a real variable, then $f(x) = a_0 + a_1x + a_2x_2^2 + \ldots\ldots + a_nx_n^n$ is called real polynomial of real variable x with real coefficients.

eg. - $3x^3 - 4x^2 + 5x - 4$, $x^2 - 2x + 1$ etc. are real polynomials.

**1.2 Complex Polynomial :** If $a_0, a_1, a_2 \ldots a_n$ be complex numbers and x is a complex variable, then $f(x) = a_0 + a_1x + a_2x^2 + a_nx^n$ is called a complex polynomial of complex variable x with complex coefficients.

eg. - $3x^2 - (2 + 4i)x + (5i - 4)$, $x^3 - 5ix^2 + (1 + 2i)x + 4$ etc. are complex polynomials.

**1.3 Degree of Polynomial :** Highest Power of variable x in a polynomial is called as a degree of polynomial

e.g. - $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots\ldots + a_{n-1}x^{n-1} + a_nx^n$ is n degree polynomial.

$f(x) = 4x^3 + 3x^2 - 7x + 5$ is 3 degree polynomial

$f(x) = 3x - 4$ is single degree polynomial or Linear polynomial

$f(x) = bx$ is odd Linear polynomial

$f(x) = x + \frac{1}{x}$ is not a polynomial in x.

**Note :** As a special case a constant is also called Polynomial of degree zero.

**1.4 Quadratic Polynomial :** A polynomial of the form $ax^2 + bx + c$( $a \neq 0$) is called a quadratic polynomial or quadratic expression in x.

e.g - $3x^2 + 7x + 5$, $x^2 - 7x + 3$

**General form :-** $f(x) = ax^2 + bx + c$

where $a, b, c \in R$ and $a \neq 0$.

## QUADRATIC EQUATION

An equation in the form of $ax^2 + bx + c = 0$, $a \neq 0$ is called quadratic equation.

e.g. - $f(x) = x^2 - 3x + 2 = 0$ is a quadratic equation as it is satisfied for $x = 2$ and 1 only.

# Quadratic Equations

## TYPES OF THE QUADRATIC EQUATION

**1.** **Complete quadratic equation :** If a, b, c are non zero constants then $ax^2 + bx + c = 0$ is called complete quadratic equation.

e.g. - (i) $3x^2 + 2x + 1 = 0$

(ii) $x^2 + 4x + 3 = 0$

**2.** **Incomplete quadratic equation :** If either b = 0 or c = 0, both are zero is called a Incomplete quadratic equation.

e.g. - (i) $2x^2 + 3 = 0$

(ii) $x^2 + 5x = 0$

(iii) $\frac{x^2}{4} + 4 = 0$

**3.** **Pure quadratic equation :** If b = 0 then It is called pure quadratic equation.

e.g. - (i) $2x^2 + 3 = 0$

(ii) $x^2 - 5 = 0$

**4.** **Adfected quadratic equation :** If $b \neq 0$, C may or may not be zero then It is called adfected quadratic equation.

e.g. - (i) $x^2 - 3x = 0$

(ii) $x^2 - 6x + 5 = 0$

## ROOTS OF AN EQUATION

We have $ax^2 + bx + c = 0$ or $a^2x^2 + abx + ac = 0$

or $\left(ax + \frac{1}{2}b\right)^2 = \frac{1}{4}(b^2 - 4ac)$ or $ax + \frac{1}{2}b = \pm\frac{1}{2}\sqrt{b^2 - 4ac}$.

$\therefore$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If $\alpha$ and $\beta$ be the roots of the equation, and $\alpha > \beta$, then

$\therefore$ $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Sum of the roots = $\alpha + \beta = -\frac{b}{a}$ and Product of the roots = $\alpha\beta = \frac{c}{a}$ and the quantity $b^2 - 4ac = \Delta$ is called discriminant of the equation.

### EXAMPLE BASED ON ROOTS OF AN EQUATION

**Ex.1** The number of real roots of the equation $|x^2 - x - 6| = x + 2$ is

(a) 1 (b) 2 (c) 3 (d) 4

**Sol.** $|x^2 - x - 6| = x + 2$

The following equation we can write

$(x^2 - x - 6) = x + 2$ and $-(x^2 - x - 6) = x + 2$.

$\Rightarrow$ $x^2 - 2x - 8 = 0$ and $x^2 - 4 = 0$

$\Rightarrow$ $(x - 4)(x + 2) = 0$ and $(x - 2)(x + 2) = 0$

$x = 4, -2$ and $x = 2, -2$

Correct answer is (C).

**Ex.2** If one root of the quadratic equation $ax^2 + bx + c = 0$ be $n^{th}$ root of the other root then

$$(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} =$$

(a) $b^{\frac{1}{n+1}}$ (b) $b^{\frac{1}{n}}$ (c) $b$ (d) $-b$

**Sol.** Let one root be $\alpha$ and other $\beta$ such that

$\beta^{1/n} = \alpha$ $\Rightarrow$ $\beta = \alpha^n$

Now, $\alpha + \beta = -\frac{b}{a}$ $\Rightarrow$ $\alpha + \alpha^n = -\frac{b}{a}$ ....... (1)

$\alpha\beta = \frac{c}{a}$ $\Rightarrow$ $\alpha\alpha^n = \frac{c}{a}$ $\Rightarrow$ $\alpha^{n+1} = \frac{c}{a}$ ....... (2)

$$\alpha = \left(\frac{c}{a}\right)^{\frac{1}{n+1}} + \left(\frac{c}{a}\right)^{\frac{n}{n+1}} = -\frac{b}{a}$$

$\Rightarrow$ $(a^n C)^{\frac{1}{n+1}} + (aC^n)^{\frac{1}{n+1}} = -b$

Hence the correct answer is (d).

**Ex.3** If the roots of the equation $ax^2 + bx + c = 0$ are of the form $\frac{K+1}{K}$ and $\frac{K+2}{K+1}$ then $b^2 - 4ac =$

(a) $(a - b + c)^2$ (b) $(a + b + c)^2$ (c) $(b + c - a)^2$ (d) $(a + b - c)^2$

**Sol.**

$$S = \frac{K+1}{K} + \frac{K+2}{K+1} = -\frac{b}{a}$$

$$P = \frac{K+1}{K} \cdot \frac{K+2}{K+1} = \frac{c}{a}$$

$\Rightarrow$ $\frac{K+2}{k} = \frac{c}{a}$ $\Rightarrow$ $K = \frac{2a}{c-a}$

Putting value of K

$$\frac{c+a}{2a} + \frac{2c}{c+a} = -\frac{b}{a}$$

$b^2 - 4ac = (a + b + c)^2$

Hence the correct answer is (b).

**Ex.4** If $\alpha$, $\beta$ are the roots of equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$ and $aS_{n+1} + cS_{n-1} =$

(a) $bS_n$ (b) $b^2S_n$ (c) $2bS_n$ (d) $-bS_n$

**Sol.** Hence $\alpha$, $\beta$ are roots.

$a\alpha^2 + b\alpha + c = 0$ .......... (1)

$a\beta + b\beta + c = 0$ .......... (2)

from (1), (2)

$aS_{n+1} + bS_n + cS_{n-1}$

$= a[\alpha^{n+1} + \beta^{n+1}] = b[\alpha^n + \beta^n] + c(\alpha^{n-1} + \beta^{n-1})$

$= [a\alpha^{n+1} + b\alpha^n + c\alpha^{n-1}] + [a\beta^{n+1} + b\beta^n + c\beta^{n-1}]$

$= 0 + 0 = 0.$

Hence $aS_{n+1} + cS_{n-1} = -bS_n$.

Hence the correct answer is (d).

**Ex.5** The roots of the equation $ix^2 - x + 12i = 0$ is

(a) $-4i, 3i$ (b) $3i, 3i$ (c) $-4i, -4i$ (d) $4i, -3i$

**Sol.** The given equation is $ix^2 - x + 12i = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4i\,(12i)}}{2i}$$

$$x = \frac{1 \pm 7}{2i}$$

$$x = \frac{4}{i}, -\frac{3}{i}$$

$$x = \frac{4i}{i^2}, -\frac{3i}{i^2}$$

$x = -4i, 3i$

Hence the correct answer is (a).

**Ex.6** The roots of the equation $\sqrt{x^2 + 3x + 32} + \sqrt{x^2 + 3x + 5} = 9$ is

(a) $-4, -1$ (b) $4, 1$ (c) $-4, 1$ (d) $4, -1$

**Sol.** The given equation is

$$\sqrt{x^2 + 3x + 32} + \sqrt{x^2 + 3x + 5} = 9$$

$\Rightarrow$ $\sqrt{x^2 + 3x + 32} = 9 - \sqrt{x^2 + 3x + 5}$

squaring both sides, we get

$x^2 + 3x + 32 = 81 + x^2 + 3x + 5 - 18\sqrt{x^2 + 3x + 5}$

$\Rightarrow$ $18\sqrt{x^2 + 3x + 5} = 54$

$\Rightarrow$ $\sqrt{x^2 + 3x + 5} = 3,$

on squaring we obtain

$\Rightarrow$ $x^2 + 3x + 5 = 9$

$\Rightarrow x^2 + 3x - 4 = 0$

$\Rightarrow (x + 4)(x - 1) = 0$

$\Rightarrow x = -4, 1$

Hence the correct answer is (c).

## PROBLEM BASED ON ROOTS OF EQUATION

**Q.1** If the sum of two roots of the equation $x^3 - px^2 + qx - r = 0$ is zero, then pq =

(a) $r$ (b) $r^2$ (c) $r^3$ (d) 0

**Q.2** The roots of the equation $x^3 - px^2 + qx - r = 0$ may be in A.P. when

(a) $2p^3 - 9pq + 27r = 0$ (b) $p^3 - 9pq + 27r = 0$

(c) $2p^3 + 9pq + 27r = 0$ (d) $p^3 - 9pq - 27r = 0$

**Q.3** If $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d$, then real root of $ax^3 + bx^2 + cx + d = 0$

(a) $-\frac{d}{a}$ (b) $\frac{d}{a}$ (c) $\frac{a}{d}$ (d) $-\frac{a}{d}$

**Q.4** In a Traingle PQR, $\angle R = \frac{\pi}{2}$, If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$, then **[AIEEE-2k5]**

(a) $a + b = c$ (b) $b + c = 0$ (c) $a + b = b$ (d) $b = c$

**Q.5** If tan A and tan B are the roots of the quadratic equation $x^2 - px + q = 0$, then value of $\sin^2 (A + B)$ is

(a) $\frac{p^2}{p^2 + q^2}$ (b) $\frac{p^2}{(q + p)^2}$ (c) $1 - \frac{p^2}{(1 - q)^2}$ (d) $\frac{p^2}{(1 - q)^2 + p^2}$

**Q.6** The number of real roots of $(x + 3)^4 + (x + 5)^4 = 16$ is

(a) 0 (b) 2 (c) 4 (d) None of these

**ANSWERS :**

**1.** (a) **2.** (a) **3.** (a) **4.** (a) **5.** (d) **6.** (b)

## NUMBER OF ROOTS

1. A quadratic equation cannot have more than two roots.
2. If there are more values of the unknown quantity which satisfies the quadratic equation. Then It is not longer an equation but an identity.

e.g. - $\frac{a^2(x-b)(x-c)}{(a-b)(a-c)} + \frac{b^2(x-c)(x-a)}{(b-c)(b-a)} + \frac{c^2(x-a)(x-b)}{(c-a)(c-b)} = x^2$

is an identity since it is satisfied by x = a, b, c.

e.g. - $x^2 = 9 = (x - 3)(x + 3)$ is an identity since it is satisfied by every value of x.

## NATURE OF ROOTS

1. When coefficients a, b, c of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, are real, then
   (i) If $D > 0$, then roots are real and unequal.
   (ii) If $D = 0$, then roots are real and equal.
   (iii) If $D < 0$, then roots are complex and conjugate of each other.

2. When coefficients a, b, c of a quadratic equation $ax^2 + bc + c = 0$, $a \neq 0$, are rational, then
   (i) If $D = 0$, then roots are rational and equal
   (ii) If $D > 0$, and D is a perfect square then roots are rational and distinct.
   (iii) If $D > 0$ and D is not a perfect square, then roots are irrational and unequal and they form pair as $p + \sqrt{q}$, $p - \sqrt{q}$; $p, q \in a$, $q > 0$.
   (iv) If $D < 0$, then roots are complex and conjugate of each other.

3. If one root of a quadratic equation with rational coefficient is $p + \sqrt{q}$ then other root is $p - \sqrt{q}$.

## EXAMPLE BASED ON NATURE OF ROOTS

**Ex.1** If $\ell$, m, n are real and $\ell \neq m$, the root of the equation $(\ell - m)x^2 - 5(\ell + m)x - 2(\ell - m) = 0$ are?

(a) Real and Equal (b) Complex (c) Real and Unequal (d) None of these

**Sol.** $B^2 - 4AC = 25(\ell + m)^2 + 8(\ell - m)^2 > 0$

correct answer is (c).

**Ex.2** Both the root of the following equation are always :

$(x - b)(x - c) + (x - a)(x - c) + (x - a)(x - b) = 0$

(a) Positive (b) Negative (c) Real (d) None of these

**Sol.** $3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0$

$\therefore B^2 - 4AC$

$= 4(a + b + c)^2 - 12(ab + bc + ca)$

$= 4[(a + b + c)^2 - 3(ab + bc + ca)] > 0$ $[\because a^2 + b^2 + c^2 > ab + bc + ca]$

$\Rightarrow$ The root are real.

Correct answer is (c).

**Ex.3** If p, q, r be in H.P. and P and r be different having same sign, then the root of the equation $px^2 + 2px + r = 0$ will be

(a) Real (b) Equal (c) Imaginary (d) None of these

**Sol.** Here p, q, r H.P. $\Rightarrow q = \frac{2pr}{p+r}$ ...... (1)

Now, $D = 4q^2 - 4pr$

$$= -4\left[Pr - \left(\frac{2pr}{P+r}\right)^2\right] \quad \text{(using (1))}$$

$$= -(Pr)\left[2\left(\frac{p-r}{P+r}\right)^2\right]$$

Since Pr > 0, P $\neq$ r given, D $\neq$ 0 and D < 0. Hence the roots are imaginary.

Hence correct answer is (c).

**Ex.4** If p, q, r, s $\in$ R and $\alpha$, $\beta$ are riits if $x^2 + px + q = 0$ and $\alpha^4$, $\beta^4$ are the roots of $x^2 + rx + 5 = 0$, then equation $x^2 - 4qx + (2q^2 - r) = 0$ has

(a) Both real roots (b) Both negative roots

(c) Both positive roots (d) one positive and second negative

**Sol.** Here $\alpha + \beta = -p$

$\alpha\beta = q$

$\alpha^4 + \beta^4 = r$

$\alpha^4\beta^4 = s$

D is $x^2 - 4qx + (2q^2 - r) = 0$ for given equation

$D = 16q^2 - 4(2q^2 - r)$

$= 18q^2 + 4r$

$= 8\alpha^2\beta^2 + 4\alpha^4 + \beta^4$

$= 4(\alpha + \beta^2)^2 > 0$

Hence given equation has real roots.

Hence correct answer is (a).

**Ex.5** Let a, b, c are non zero number then $\int_0^1 (1+\cos^8 x)(ax^2 + bx + c)\,dx = \int_0^2 (1+\cos^8 x)(ax^2 + bx + c)$ dx then quadratic equation $ax^2 + bx + c = 0$

(a) No roots in (0, 2) (b) One root atleast is (1, 2)

(c) Both roots in (0, 2) (d) Both roots imaginary

**Sol.** Here function is differentiable (and continuous)

Hence $\phi(1) = \phi(2) \Rightarrow \phi'[(K)] = 0$ in interval (0, 2)

$\therefore \quad (1 + \cos^8 K)(aK^2 + bk + c) = 0$

$\Rightarrow aK^2 + bK + c = 0$

in this way atlest one root of equation $ax^2 + bx + c = 0$ has exist.

Hence the correct answer is (b).

**Ex.6** The roots of quadratic equation are always $(b - c)x^2 + (c - a)x + (a - b) = 0$

(a) Positive (b) Rational (c) Neagative (d) None of these

**Sol.** $\Delta = (c - a)^2 - 4(b - c)(a - b)$

$\Rightarrow c^2 + a^2 - 2ac - 4(ab - b^2 - ac + bc)$

$\Rightarrow c^2 + a^2 + 2ac + 4b^2 - 4ab - 4bc$

$\Rightarrow (c + a)^2 + (2b)^2 - 4b(a + c)$

$\Rightarrow (c + a)^2 + (2b)^2 - 2 \cdot 2b(c + a)$

$\Rightarrow (c + a - 2b)^2$

$\Delta$ is square of a rational number.

Hence the correct answer is (b).

## PROBLEM BASED ON NATURE OF ROOTS

**Q.1** If $\cos\theta \cdot \sin\phi$, $\sin\theta$ are in G.P., then roots of $x^2 + 2\cot\phi \cdot x + 1 = 0$ are always

(a) equal (b) real (c) imaginary (d) greater than 1.

**Q.2** If $a \in R$, $b \in R$ then the equation $x^2 - abx - a^2 = 0$ has

(a) one positive root and one negative root (b) both positive roots

(c) both negative roots (d) no real roots.

**Q.3** Let $a > 0$, $b > 0$ and $c > 0$, then both the roots of the equation $ax^2 + bx + c = 0$

(a) are real and negative (b) have negative real parts

(c) are rational numbers (d) None of these

**Q.4** If $(2x^2 - 3x + 1)(2x^2 + 5x + 1) = 9x^2$ then equation has

(a) four real roots (b) two real and two imaginary roots

(c) all imaginary (d) None of the above

**Q.5** If $(x + 2)(x + 3)(x + 8)(x + 12) = 4x^2$ then equation has

(a) no real roots (b) all real roots

(c) can't be disscussed (d) None of these

**Q.6** If $(x - 2)^6 + (x - 4)^6 = 64$, then equation has

(a) real and irrational roots (b) real and rational roots

(c) two real and two irrational roots (d) None of these

**Q.7** If $(12x - 1)(6x - 1)(4x - 1)(3x - 1) = 5$ then equation has

(a) real and irrational roots (b) real and rational roots

(c) two real and two irrational roots (d) None of these

**Q.8** The condition that the equation $\frac{1}{x} + \frac{1}{x+b} = \frac{1}{m} + \frac{1}{m+b}$

has real roots that are equal in magnitude but opposite in sign is

(a) $b^2 = m^2$ (b) $b^2 = 2m^2$ (c) $2b^2 = m^2$ (d) None of these

**ANSWERS :**

**1.** (b) **2.** (a) **3.** (b) **4.** (a) **5.** (b) **6.** (d)
**7.** (d) **8.** (b)

## RELATION BETWEEN ROOTS AND COEFFICIENTS

If $\alpha$ and $\beta$ are two roots of the equation $ax^2 + bx + c = 0$, then

(i) Sum of the roots $\alpha + \beta = -\frac{b}{a} = -\frac{\text{coeff. of } x}{\text{coeff. of } x^2}$

(ii) Product of the roots $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coeff. of } x^2}$

**Symmetric functions of the roots**

A function of the roots $\alpha$ and $\beta$ is called symmetric if it remains unaltered on interchanging $\alpha$ and $\beta$.

(a) $\alpha^2 - \beta^2 = [(\alpha + \beta)^2 - 2\alpha\beta]$

(b) $\alpha - \beta = [(\alpha + \beta)^2 - 4\alpha\beta]^{1/2}$

(c) $\alpha^3 - \beta^3 = [(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)]$

(d) $\alpha^4 - \beta^4 = [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2\alpha^2\beta^2$

## EXAMPLE BASED ON RELATION BETWEEN ROOTS AND COEFFICIENTS

**Ex.1** If the sum of roots of the quadratic equation $ax^2 + bx + c = 0$, $(a, b, c \neq 0)$ is equal to sum of the squares of their reciprocals then $\frac{a}{c}, \frac{b}{a}, \frac{c}{a}$ are in **[AIEEE-2k3]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

**Sol.** $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$$

$$\Rightarrow \quad -\frac{b}{a} = \frac{b^2 - 2ac}{c^2}$$

$$\Rightarrow \quad \frac{2a}{c} = \frac{b^2}{c^2} + \frac{b}{a} = \frac{ab^2 + bc^2}{ac^2}$$

$$\Rightarrow \quad 2a^2c = ab^2 + bc^2$$

$$\Rightarrow \quad \frac{2a}{b} = \frac{b}{c} + \frac{c}{a}$$

$\frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P. and $\frac{a}{c}, \frac{b}{a}, \frac{c}{a}$ are in H.P.

Hence the correct answer is (c).

**Ex.2** If $\alpha, \beta, \gamma$ are such that $\alpha + \beta + \gamma = 2$, $\alpha^2 + \beta^2 + \gamma^2 = 6$, $\alpha^3 + \beta^3 + \gamma^3 = 8$, then $\alpha^4 + \beta^4 + \gamma^4 = ?$

(a) 5 (b) 18 (c) 12 (d) 36

**Sol.** $(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\beta\gamma + \gamma\alpha + \alpha\beta)$

$\Rightarrow 4 = 6 + 2(\beta\gamma + \gamma\alpha + \alpha\beta)$

$\Rightarrow \beta\gamma + \gamma\alpha + \alpha\beta = -1$

Also, $\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma$

$(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \beta\gamma - \gamma\alpha - \alpha\beta)$

$\Rightarrow 8 - 3\alpha\beta\gamma = 2(6 + 1)$

$\Rightarrow 3\alpha\beta\gamma = 8 - 14 = -6$ or $\alpha\beta\gamma = -2$

Now, $(\alpha^2 + \beta^2 + \gamma^2)^2 = \Sigma\alpha^4 + 2\Sigma\beta^2\gamma^2$

$= \Sigma\alpha^4 + 2[(\Sigma\beta\gamma)^2 - 2\alpha\beta\gamma(\Sigma\gamma)]$

$\Rightarrow \Sigma\alpha^4 = 36 - 2[(-1)^2 - 2(-2)(2)]$

$\Rightarrow \Sigma\alpha^4 = 18$

Hence (b) is correct answer.

**Ex.3** If $\alpha, \beta, \gamma$ be the roots of the equation $x(1 + x^2) + x^2(6 + x) + 2 = 0$, then the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ is

(a) $-3$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) None of these

**Sol.** $2x^3 + 6x^2 + x + 2 = 0$ has roots $\alpha, \beta, \gamma$

So, $2x^3 + x^2 + 6x + 2 = 0$ has roots $\alpha^{-1}, \beta^{-1}, \gamma^{-1}$

Hence sum of the roots $= -\dfrac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$

$\alpha^{-1} + \beta^{-1} + \gamma^{-1} = -\frac{1}{2}$

Hence the correct answer is (c).

**Ex.4** If $\alpha, \beta$ are the roots of $x^2 + px + q = 0$ and also of $x^{2n} + p^n x^n + q^n = 0$ and if $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are roots of $x^n + 1 + (x + 1)^n = 0$ then n is

(a) An integer (b) An odd integer (c) An even integer (d) None of these

**Sol.** Here $\alpha + \beta = -p$

$\alpha\beta = q$

Also, $\begin{cases} \alpha^{2n} + p^n\alpha^n + q^n = 0 \\ \beta^{2n} + p^n\beta^n + q^n = 0 \end{cases}$

$\Rightarrow (\alpha^{2n} - \beta^{2n}) + p^n(\alpha^n - \beta^n) = 0$

$\Rightarrow \alpha^b + \beta^n = -p^n$

again $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ are roots of $x^n + 1 + (x + 1)^n = 0$

$\Rightarrow \left(\frac{\alpha}{\beta}\right)^n + 1 + \left(\frac{\alpha}{\beta} + 1\right)^n = 0$

$\Rightarrow \alpha^n + \beta^n + (\alpha + \beta)^n = 0$

$\Rightarrow -p^n + (-p)^n = 0$ which is true only if n is even integer.

Hence the correct answer is (c).

**Ex.5** If $\alpha$, $\beta$ are roots of the equation $2x^2 + 6x + b = 0$ $(b < 0)$ then $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is less than

(a) 2 (b) –2 (c) 18 (d) None of these

**Sol.** $\alpha + \beta = -3$, $\alpha\beta = \frac{b}{2}$ since $b < 0$,

this discriminant $D = 36 - 4b > 0$ so $\alpha$ and $\beta$ are real.

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} - 2 = \frac{18}{b} - 2$$

$$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} < -2.$$

Hence the correct answer is (b).

## PROBLEM BASED ON RELATION BETWEEN ROOTS AND COEFFICIENTS

**Q.1** If $\alpha$, $\beta$ are roots of the equation $ax^2 + 3x + 2 = 0$, $(a < 0)$, then $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is greater than

(a) 0 (b) 1 (c) 2 (d) None of these

**Q.2** The value of a for which the sum of the square of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume least value is

(a) 0 (b) 1 (c) 2 (d) 3

**Q.3** Let $\alpha$, $\beta$ be the roots of $ax^2 + bx + c = 0$, $\gamma$, $\delta$ be the roots of $px^2 + qx + r = 0$ and $D_1$, $D_2$ the respective discriminants of these equations. If $\alpha$, $\beta$, $\gamma$ and $\delta$ are in A.P. then $D_1 . D_2 =$

(a) $\frac{a^2}{b^2}$ (b) $\frac{a^2}{p^2}$ (c) $\frac{b^2}{q^2}$ (d) $\frac{c^2}{r^2}$

**Q.4** If $\alpha$, $\beta$ are roots of $ax^2 + bx + c = 0$ and $\alpha + h$, $\beta + h$ are the roots of the $px^2 + qx + r = 0$ and $D_1$, $D_2$ the respective discriminants of these equations, then $D_1 : D_2 =$

(a) $\frac{a^2}{p^2}$ (b) $\frac{b^2}{q^2}$ (c) $\frac{c^2}{r^2}$ (d) None of these

**Q.5** If $\alpha$, $\beta$ are the roots of $ax^2 + bx + c = 0$ and $\alpha + h$, $\beta + h$ are the roots of $px^2 + qx + r = 0$ then h = ?

(a) $\left(\frac{b}{a} - \frac{q}{p}\right)$ (b) $\frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$ (c) $-\frac{1}{2}\left(\frac{b}{a} - \frac{p}{q}\right)$ (d) None of these

**Q.6** If even pair from among the equations $x^2 + px + qr = 0$, $x^2 + qx + rp = 0$ and $x^2 + rx + pq = 0$ has a common root, then the sum of the the three common roots is

(a) $2(p + q + r)$ (b) $p + q + r$ (c) $-(p + q + r)$ (d) $p q r$

**ANSWERS :**

**1.** (d) **2.** (b) **3.** (b) **4.** (a) **5.** (b) **6.** (b)

# Quadratic Equations

## FORAMATION OF A QUADRATIC EQUATION

The quadratic equation whose roots $\alpha$, $\beta$ are known is given by

$$(x - \alpha)(x - \beta) = 0$$

or $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e. $x^2$ – (sum of the roots) x + (product of roots) = 0.

## EXAMPLE BASED ON FORMATION OF A QUADRATIC EQUATION

**Ex.1** If $\alpha$, $\beta$ are the root of a quadratic equation $x^2 - 3x + 5 = 0$ then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is ?

(a) $x^2 + 4x + 4 = 0$ (b) $x^2 - 4x + 4 = 0$ (c) $x^2 - 4x - 4 = 0$ (d) $x^2 + 4x - 4 = 0$

**Sol.** Since $\alpha$, $\beta$ are the roots of equation

$$x^2 - 3x + 5 = 0$$

So, $\alpha^2 - 3\alpha + 5 = 0$ $\beta^2 - 3\beta + 5 = 0$

$\therefore$ $\alpha^2 - 3\alpha = -5$ $\beta^2 - 3\beta = -5$

Putting in $(\alpha^2 - 3\alpha + 7)$ & $(\beta^2 - 3\beta + 7)$

$-5 + 7$ & $-5 + 7$

2 & 2

Sum of roots $2 + 2 = 4$.

Product of roots $2 \cdot 2 = 4$

The requried eq. is $x^2 - 4x + 4 = 0$.

**Ex.2** If a and b are rational and b is not a perfect square, then the quadratic equation with rational coefficient whose one root is $\frac{1}{a+\sqrt{b}}$ is

(a) $x^2 - 2ax + (a^2 - b) = 0$ (b) $(a^2 - b)x^2 - 2ax + 1 = 0$

(c) $(a^2 - b)x^2 - 2bx + 1 = 0$ (d) None of these

**Sol.** As irrational roots always occurs in pairs i.e. the roots of given equation are

$$\frac{1}{a+\sqrt{b}}, \frac{1}{a-\sqrt{b}}$$

Thus sum of roots = $\frac{2a}{a^2 - b}$

and product of roots = $\frac{1}{a^2 - b}$

Hence, the quadratic equation

$$x^2 - \left(\frac{2a}{a^2-b}\right)x + \left(\frac{1}{a^2-b}\right) = 0$$

$$(a^2 - b)x^2 - 2ax + 1 = 0$$

Hence (b) is correct Answer.

**Ex.3** The quadratic equation whose roots are the A.M. and H.M. of the roots of the equation $x^2 + 7x - 1 = 0$ is

(a) $14x^2 + 14x - 45 = 0$ (b) $14x^2 - 14x + 14 = 0$

(c) $14x^2 + 45x - 14 = 0$ (d) None of these

**Sol.** Here $x^2 + 7x - 1 = 0$ has $\alpha + \beta = -7$ and $\alpha\beta = -1$

$\therefore$ quadratic whose roots are A.M. and H.M. of the roots of the equation $x^2 + 7x - 1 = 0$

$\Rightarrow \quad \alpha' = \frac{\alpha+\beta}{2}$ and $\beta' = \frac{2\alpha\beta}{\alpha+\beta}$

or $\alpha' = -\frac{7}{2}$ and $\beta' = \frac{-2}{-7} = \frac{2}{7}$.

required eq is, $x^2 - (\alpha' + \beta')x + \alpha'\beta' = 0$

or $x^2 - \left(-\frac{45}{14}x\right) - 1 = 0$

or $14x^2 + 45x - 14 = 0$

Hence the correct answer is (c).

**Ex.4** The equation whose roots are cubes of the roots of the equation $ax^3 + bx^2 + cx + d = 0$ is

(a) $a^3x^3 - x^2 (3a^2d - 3abc + b^3) + x (3ad^2 + 3bcd + c^3) + d^3 = 0$

(b) $a^3x^3 + x^2 (3a^2d - 3abc + b^3) + x (3ad^2 - 3bcd + c^3) + d^3 = 0$

(c) $a^3x^3 + x^2 (3a^2d - 3abc + b^3) - x (3ad + 3bcd + c^3) + d^3 = 0$

(d) $a^3x^3 + x^2 (3a^2d + 3abc + b^3) + x (3ad^2 + 3bcd + c^3) + d^3 = 0$

**Sol.** The given equation is $ax^3 + bx^2 + cx + d = 0$ replacing x by $x^{1/3}$ we get

$a(x^{1/3})^3 + b(x^{1/3})^2 + c(x^{1/3}) + d = 0$

$\Rightarrow ax + d = -(bx^{2/3} + cx^{1/3})$

$\Rightarrow (ax + d)^3 = -(bx^{2/3} + cx^{1/3})^3$

$\Rightarrow a^3x^3 + 3a^2d^2x^2 - 3ad^2x + d^3 = -\{b^3x^2 + c^3x + 3bcx (bx^{2/3} + cx^{1/3})\}$

$\Rightarrow a^3x^3 + 3a^2dx^2 - 3ad^2x + d^3 = -\{b^3x^2 + c^3x + 3bcx (ax + d)\}$

$\Rightarrow a^3x^3 + x^2 (3a^2d - 3abc + b^3) + x (3ad^2 - 3bcd + c^3) + d^3 = 0$

Hence the correct answer is (b).

## PROBLEM BASED ON FORMATION OF A QUADRATIC EQUATION

**Q.1** The quadratic equation whose roots are A.M. and H.M. between the roots of the equation $ax^2 + bx + c = 0$ is

(a) $abx^2 + (b^2 + ac)x + bc = 0$ (b) $2abx^2 + (b^2 + 4ac)x + 2bc = 0$

(c) $2abx^2 + (b^2 + ac)x + bc = 0$ (d) None of these

**Q.2** Let $\Delta^2$ be the discriminate and $\alpha, \beta$ be the roots of the equation $ax^2 + bx + c = 0$. Then $2a\alpha + \Delta$ and $2a\beta - \Delta$ can be the roots of the equation

(a) $x^2 + 2bx + b^2 = 0$ (b) $x^2 - 2bx + b^2 = 0$

(c) $x^2 + 2bx - 3b^2 + 16ac = 0$ (d) both b and c.

**Q.3** Let $a = \cos\frac{2\pi}{7} + i \sin\frac{2\pi}{7}$, $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$. Then the equation whose roots are $\alpha, \beta$ is

(a) $x^2 - x + 2 = 0$ (b) $x^2 + x - 2 = 0$ (c) $x^2 - x - 2 = 0$ (d) $x^2 + x + 2 = 0$

**ANSWERS :**

**1.** (b) **2.** (d) **3.** (d)

## CONDITION FOR COMMON ROOTS

Let $a_1x^2 + b_1x + c_1 = 0$ ....... (1)

and $a_2x^2 + b_2x + c_2 = 0$ ....... (2)

be two given equation.

## FOR ONE ROOT COMMON

Let $\alpha$ be a common root of (1) and (2)

$a_1\alpha^2 + b_1\alpha + c_1 = 0$ ....... (1)

$a_2\alpha^2 + b_2\alpha + c_2 = 0$ ....... (2)

$$\therefore \quad \frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_1c_1)\,(a_1b_2 - a_2b_1)$$

## FOR BOTH ROOTS COMMON

When (1) and (2) have both roots common then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

## EXAMPLE BASED ON COMMON ROOTS

**Ex.1** If the equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have one root common and $p \neq q$, then $p + q =$

(a) 1 (b) −1 (c) 0 (d) 2

**Sol.** Common root of the equation $\alpha$ is, then

$$\frac{\alpha^2}{p^2 - q^2} = \frac{\alpha}{q - p} = \frac{1}{q - p}$$

$\Rightarrow \quad \alpha = -(p + q) = 1$

$\Rightarrow \quad p + q = -1$

Correct answer is (b).

**Ex.2** If equation $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have one root common, then the value of k is

(a) −3 (b) $-\frac{27}{4}$ (c) both a and b (d) None of these

**Sol.** Common root of the equation $\alpha$ is

then $$\frac{\alpha^2}{-4K - 15} = \frac{\alpha}{-5 + 8} = \frac{1}{-6 - k}$$

$$\alpha^2 = \frac{4K + 15}{K + 6} \text{ and } \alpha = \frac{-3}{K + 6}$$

$$= \left(\frac{-3}{K+6}\right)^2 = \frac{4K+15}{K+6}$$

$\Rightarrow$ (4K + 15) (K + 6) = 9

$\Rightarrow$ $4K^2 + 39K + 81 = 0$

$\Rightarrow$ $K = -3$ or $K = -\frac{27}{4}$.

Hence the correct answer is (c).

**Ex.3** If $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$ are in A.P. and the equation $a_1x^2 + 2b_1x + c_1 = 0$ and $a_2x^2 + 2b_2x + c_2 = 0$ have a common root, then $a_2$, $b_2$, $c_2$ are in

(a) G.P. (b) A.P. (c) H.P. (d) None of these

**Sol.** Let $\alpha$ be the common root :

$$\frac{\alpha^2}{2(b_1c_2 - b_2c_1)} = \frac{\alpha}{c_1a_2 - c_2a_1} = \frac{1}{2(a_1b_2 - a_2b_1)}$$

$\Rightarrow$ $\alpha^2 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $\alpha = \frac{c_1a_2 - c_2a_1}{2(a_1b_2 - a_2b_1)}$

Now, $\alpha^2 = (\alpha)^2$

$\Rightarrow$ $\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} = \left\{\frac{c_1a_2 - c_2a_1}{2(a_1b_2 - a_2b_1)}\right\}^2$

$\Rightarrow$ $4\,(b_1c_2 - b_2c_1)\,(a_1b_2 - a_2b_1) = (c_1a_2 - c_2a_1)^2$

$\Rightarrow$ $4\, b_2c_2 \left(\frac{b_1}{b_2} - \frac{c_1}{c_2}\right) .\ a_2b_2 \left(\frac{a_1}{a_2} - \frac{b_1}{b_2}\right)$

$\Rightarrow$ $(c_2a_2)^2 - \left(\frac{c_1}{c_2} - \frac{a_1}{a_2}\right)^2$

$\Rightarrow$ $4\, a_2b_2^{\,2}c_2 \left(\frac{b_1}{b_2} - \frac{c_1}{c_2}\right)\left(\frac{a_1}{a_2} - \frac{b_1}{b_2}\right)$

$\Rightarrow$ $a_2^{\,2}c_2^{\,2} \left(\frac{c_1}{c_2} - \frac{a_1}{a_2}\right)^2$

It is given that $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$ are in A.P.

Let D be the common difference of this A.P. then

$$\frac{b_1}{b_2} - \frac{a_1}{a_2} = \frac{c_1}{c_2} - \frac{b_1}{b_2} = D$$

and $\frac{c_1}{c_2} - \frac{a_1}{a_2} = 2D$

Putting these value in (i) we get

$4a_2b_2^2c_2x\ (-D) \times (-D) = a_2^2c_2^2(2D)^2$

$\Rightarrow\ b_2^2 = a_2c_2 \qquad \Rightarrow\ a_2,\ b_2,\ c_2$ are in G.P.

Hence the correct Answer is (a).

## PROBLEM BASED ON COMMON ROOTS

**Q.1** If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$, $a \neq 0$, $b \neq 0$ have a common root, then $a^3 + b^3 + c^3 =$

(a) abc (b) $-3abc$ (c) $3abc$ (d) $a^2b^2c^2$

**Q.2** The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$

(a) 0 (b) $-2$ (c) 2 (d) both a and b

**Q.3** If the roots of $a_1x^2 + b_1x + c_1 = 0$ are $\alpha_1, \beta_1$ and those of $a_2x^2 + b_2x + c_2 = 0$ are $\alpha_2, \beta_2$ such that $\alpha_1\alpha_2 = \beta_1\beta_2 = 1$, then

(a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (b) $\frac{a_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{a_2}$ (c) $a_1a_2 = b_1b_2 = c_1c_2$ (d) None of these

**Q.4** If $x^2 - 2r\,.\,P_rx + r = 0$; $r = 1, 2, 3$, are three quadratic equations of which each pair has exactly one root common, then the number of solution of the triplet $(P_1, P_2, P_3)$ is

(a) 2 (b) 1 (c) 9 (d) 27

**Q.5** If the equation $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$ $(b \neq c)$ have a common root then $a + 4b + 4c =$

(a) 2 (b) 1 (c) 0 (d) $-1$

**ANSWERS :**

**1.** (c) **2.** (d) **3.** (b) **4.** (b) **5.** (c)

## SIGN OF QUADRATIC EXPRESSION

$ax^2 + bx + c = y$ (say)

(i) If $b^2 - 4ac = 0$ then sign of y is same as that of 'a' for all real values of x.
Since in this case $ax^2 + bx + c = a(x - \alpha)^2$

(ii) If $b^2 - 4ac < 0$, then sign of y is again same as that of a.

(iii) If $b^2 - 4ac < 0$, then sign of the quadratic expression depends upon the roots of equation $ax^2 + bx + c = 0$.

## MAXIMUM AND MINIMUM VALUES OF QUADRATIC EXPRESSION

Let $ax^2 + bx + c$ be a given expression. we can write

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(\frac{4ac - b^2}{4a}\right)$$

**Case I :** If a > 0 then the minimum value of the given expression = $\frac{4ac - b^2}{4a}$ for $x = -\frac{b}{2a}$

**Case II :** If a < 0 then the minimum value of the given expression = $\frac{4ac - b^2}{4a}$ for $x = -\frac{b}{2a}$

## EXAMPLE BASED ON MAXIMUM AND MINIMUM VALUE OF A QUADRATIC EXPRESSION

**Ex.1** The minimum value of the expression $4x^2 + 2x + 1$ is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

**Sol.** a = 4 > 0 therefore the minimum value is

$$\frac{4(4)(1) - (2)^2}{4(4)} = \frac{16 - 4}{16} = \frac{12}{16} = \frac{3}{4}$$

Hence the correct answer is (a).

**Ex.2** If x is real then maximum value of

$$y = 2(a - x)\left(x + \sqrt{x^2 + b^2}\right) \text{ is}$$

(a) $a^2 - b^2$ (b) $a^2 + b^2$ (c) $b^2 - a^2$ (d) None of these

**Sol.** Let $t = x + \sqrt{x^2 + b^2}$

$$\Rightarrow \frac{1}{t} = \frac{1}{x + \sqrt{x^2 + b^2}}$$

$$\Rightarrow = \frac{\sqrt{x^2 + b^2} - x}{b^2}$$

$$t - \frac{b^2}{t} = 2x \text{ and } t + \frac{b^2}{t} = 2\sqrt{x^2 + b^2}$$

Thus $2(a-x)(x+\sqrt{x^2+b^2}) = \left(2a-t+\frac{b^2}{t}\right)t$

$= 2at - t^2 + b^2$
$= a^2 + b^2 - (a^2 - 2at + t^2)$
$= a^2 + b^2 - (a-t)^2$

Therefore $y = 2(a-x)(x+\sqrt{x^2+b^2}) \le a^2 + b^2$.

Hence the maximum value of y is $a^2 + b^2$.

Hence the correct answer is (b).

**Ex.3** If $x^2 - 4x + \log_{1/2} a^2 = 0$ doesnot have two distinct real roots, then maximum value of a is

(a) $\frac{1}{16}$ (b) $\frac{1}{18}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

**Sol.** Since $x^2 - 4x + \log_{1/2} a^2 = 0$ doesnot have two distinct real roots, discriminant $\le 0$.

$\Rightarrow 16 - 4\log_{1/2} a^2 \ge 0$

$\Rightarrow \log_{1/2} a^2 \ge 4$

$\Rightarrow a^2 \ge \frac{1}{16}$

Hence the correct answer is (a).

**Ex.4** The difference of maximum and minimum value of $\frac{x^2+4x+9}{x^2+9}$ is

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{8}{7}$ (d) $\frac{7}{8}$

**Sol.** Let $y = \frac{x^2+4x+9}{x^2+9}$

$(y-1)x^2 - 4x + 9(y-1) = 0$

For real value of x, $D \ge 0$.

$16 - 36(y-1)^2 \ge 0$

$4 - 9(y-1)^2 \ge 0$

$\{2 - 3(y-1)\}\{2 + 3(y-1)\} \ge 0$

$(5 - 3y)(3y - 1) \ge 0$

$\frac{1}{3} \le y \le \frac{5}{3}$.

difference of maximum and minimum value is

$\frac{5}{3} - \frac{1}{3} = \frac{4}{3}$.

**Ex.5** If x be real, then maximum and minimum values of the expression $\frac{x^2 - x + 1}{x^2 + x + 1}$ are

(a) $3, -\frac{1}{3}$ (b) $-3, \frac{1}{3}$ (c) $3, \frac{1}{3}$ (d) $-3, -\frac{1}{3}$

**Sol.** Let $\frac{x^2 - x + 1}{x^2 + x + 1} = y$

$\Rightarrow x^2 (y - 1) + (y + 1)x + (y - 1) = 0$

x is real then $\Delta \geq 0$

$(y + 1)^2 - 4(y - 1)^2 \geq 0$

$\Rightarrow 3y^2 - 10y + 3 \leq 0.$

$(y - 3)(3y - 1) \leq 0$ or $y - 3 \geq 0.3y - 1 \leq 0$

$\Rightarrow y \leq 3, y \geq \frac{1}{3}$ or $y \geq 3, y \leq \frac{1}{3}$

but $y \geq 3, y \leq \frac{1}{3}$ is not possible. Hence $\frac{1}{3} \leq y \leq 3$.

Hence correct answer is (c).

## PROBLEM BASED ON MAXIMUM AND MINIMUM VALUE OF A QUADRATIC EXPRESSION

**Q.1** The minimum value of $x^2 - 2x + 10$ is

(a) 7 (b) 4 (c) 9 (d) 1

**Q.2** The maximum value of $4x - 5x^2 - 1$ is

(a) $-\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{5}$ (d) $-\frac{3}{5}$

**Q.3** For all real values of x, the maximum value of the expression $\frac{x}{x^2 - 5x + 9}$ is

(a) 1 (b) 2 (c) −1 (d) −2

**Q.4** The maximum value of $\left(\frac{1}{2}\right)^{x^2-3x+2}$ is

(a) $2^{1/4}$ (b) $2^{1/2}$ (c) $-2^{1/4}$ (d) $-2^{1/2}$

**Q.5** The maximum value of ln {(5 − x) (x − 3)}, $3 < x < 5$ is

(a) 0 (b) 1 (c) 2 (d) −1

**ANSWERS :**

**1.** (c) **2.** (a) **3.** (a) **4.** (a) **5.** (a)

# Quadratic Equations

## TIME SAVING TIPS AND SHORTCUTS

* If $\alpha$ and $\beta$ are the roots of $ax^2 + bx + c = 0$ then $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

* If sum of the coefficients of a quadratic equation is zero then one root is 1.

  i.e. If $a + b + c = 0$ then the roots of the quadratic equation are 1 and $\frac{c}{a}$ and if $a - b + c = 0$ then the roots are $-1, -\frac{c}{a}$.

* If the roots of $ax^2 + bx + c = 0$ are in the ratio m : n then condition is $mnb^2 = ac(m + n)^2$.

* If the roots are reciprocal to each either then $a = c$.

* If the roots are of opposite signs then $b = 0$.

* One root of a quadratic equation is zero if $c = 0$.

* If $a < 0$, then $\frac{4ac - b^2}{4a}$ is the maximum value of f(x).

* If $a > 0$, then $\frac{4ac - b^2}{4a}$ is the minimum value of f(x).

* If $ax^2 + bx + c$ is +ve then $b^2 - 4ac$ must be –ve and a must be +ve.

* If $ax^2 + bx + c$ is –ve then $b^2 - 4ac$ must be +ve and a must be +ve.

* If $ax^2 + bx + c$ is –ve then $b^2 - 4ac$ must be –ve and a must be –ve.

* If $ax^2 + bx + c = 0$ is satisfied by more than two roots then $a = 0, b = 0, c = 0$.

* The equation $f(x) = 0$ has two real roots $\alpha$ and $\beta$ then $f'(x) = 0$ will have a real root lying between $\alpha$ and $\beta$.

* A decrease or increase in the roots of a quadratic polynomial does not alter its maximum or minimum value.

* If $a + b + c = 0$, and a, b, c are rational then $ax^2 + bx + c = 0$ will have rational roots.

## Quadratic Equations

* If the ratio of the roots of the equation $ax^2 + bx + c = 0$ is equal to the ratio of the roots of $AX^2 + BX + C = 0$ then $\frac{b^2}{ac} = \frac{B^2}{AC}$.

* If $f(x) = ax^2 + bx + c$ be a perfect square then $D = b^2 - 4ac = 0$ and $a > 0$.

* Complex roots always occurs in conjugate pairs.

* Surd roots always occur in conjugate pairs.

* If $a > 0$, $C < 0$ or $a < 0$, $C > 0$, roots are of opposite sign.

* If $a < 0$, $b > 0$, $c > 0$ or $a > 0$, $b < 0$, $c > 0$, then both roots are positive.

* If $b = 0$, $c = 0$, both the roots are 0 and $-\frac{b}{a}$.

* If $\alpha$ . $\beta$ are the roots of $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$ then $aS_{n+1} + cS_{n-1} + b_Sn = 0$.

* If $a > 0$, $D = b^2 - 4ac > 0$ then minimum of $f(x) = ax^2 + bx + c$ is negative.