Problem Based on Kinetic Molecular Model of a Gas

*** Postulates of the model.**
*** Distribution of molecular speeds**
*** Explanation of gas laws on the basis of the kinetic molecular model**
(Boyle's Law, (ii) Charle's Law, (iii) Dalton's Law, (iv) Graham's Law)

VERY SHORT ANSWER TYPE QUESTIONS :

VSA.1 What is meant by elastic collision ?

Sol. Elastic collision is a collision in which there is no net loss of energy rather there is transfer of energy.

VSA.2 Give expression for kinetic energy of n moles of gas.

Sol. K.E. = $\frac{3}{2}$ nRT.

VSA.3 Give expression for (i) root mean square velocity, (ii) most probable velocity, (iii) average velocity.

Sol. $u = \sqrt{\frac{3RT}{M}}$, $v = \sqrt{\frac{8RT}{M\pi}}$, $\alpha = \sqrt{\frac{2RT}{M}}$

where 'u' is r.m.s., v is average velocity and 'α' is most probable velocity.

VSA.4 What is the relationship between average kinetic energy and the temperature of a gas ?

Sol. $\overline{K.E.} = \frac{3}{2}kT$

Where k is Boltzmann constant = $\left(\frac{R}{N_0}\right)$

VSA.5 Which two postulates of the kinetic molecular theory are only approximations when applied to real gases ?

Sol. (i) Intermolecular forces between molecules are negligible.

(ii) Molecules of a gas have negligible volumes.

VSA.6 What is the ratio of average molecular kinetic energy of CO_2 to that of SO_2 at 27ºC ?

Sol. Average kinetic energy depends only on temperature and not on the nature of the substance. Thus, the ratio is 1 : 1.

VSA.7 Distinguish between the total kinetic energy of a molecule and its translational kinetic energy. For what type of gas molecules are they same ?

Sol. The total kinetic energy of a molecule includes the translational kinetic energy, rotational kinetic energy and vibrational kinetic energies.

For monoatomic gases both rotational and vibrational kinetic energies are zero. Therefore, for these, total kinetic energy is equal to translational kinetic energy.

VSA.8 What would have happened of the gas pressure if the molecular collisions were not elastic ?

Sol. The gas pressure would have gradually become zero.

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SHORT ANSWER TYPE QUESTIONS :

SA.1 Calculate the root mean square velocity of oxygen molecule at 300 K. (At. wt. of O = 16 amu)

Sol. R = 8.314 J, T = 300 K, M = 32 × 10^{-3} kg mol^{-1}

$$u = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\times8.314\times300}{32\times10^{-3}kg}} = 484\ ms^{-1}.$$

SA.2 Give various postulates of kinetic theory of gases.

Sol. (i) A gas consists of a large number of identical molecules of mass m. The dimensions of these molecules are very very small compared to the space between them. Hence the molecules are treated as point masses.

(ii) There are practically no attractive forces between the molecules. The molecules therefore, move independently.

(iii) The molecules are in a ceaseless and random motion, colliding with each other and with the walls of the container. The direction of their motions change only on collision. These collisions are known as elastic collisions in which the energy and momenta of the molecules are conserved. In a non-elastic collision these quantities are not conserved.

(iv) The pressure of a gas is the result of collision of molecules with the walls of the container.

SA.3 Calculate the total and average kinetic energy of 32g of methane molecules at 27ºC, R = 8.314 JK^{-1} mol^{-1}.

Sol. $K.E. = \frac{3}{2}\ nRT = \frac{3}{2} \times \frac{32}{16} \times 8.314 \times 300\ K$

$$= \frac{3}{2} \times 2 \times 8.314 \times 300 = 2494.2 \times 3 = 7382.6\ J\ mol^{-1}$$

Average kinetic energy = $\frac{7382.6}{2\times6.022\times10^{23}} = \frac{7382.6}{12.044\times10^{23}} = 6.13 \times 10^{-21}$ J of molecules.

SA.4 Starting from kinetic gas equation, prove that the average kinetic energy of a gas is directly proportional to its absolute temperature.

Sol. **Relation between average kinetic energy and absolute temperature deduction from kinetic theory :** According to Kinetic gas equation

$$PV = \frac{1}{3}mnu^2 \qquad \text{.......... (i)}$$

Where P is the pressure, V is the volume of the gas, m is the mass of each molecule, n is the number of molecules present and u is the root mean square velocity of the molecules.

If 1 mole of the gas is taken, then the total mass of the gas, m × n = M, the molar mass of the gas. Hence eqn. (i) becomes

$$PV = \frac{1}{3}Mu^2 \qquad \text{.......... (ii)}$$

It can be re-written as $PV = \frac{2}{3}.\frac{1}{2}Mu^2$

But $\frac{1}{2}Mu^2$ represents the kinetic energy (K.E.) per mole

Hence $PV = \frac{2}{3} \times K.E.$

or $K.E. = \frac{3}{2}PV$ for 1 mole of the gas .... (iii)

But for 1 mole of the gas, the ideal gas equation is

$PV = RT$

Putting this value in eqn. (iii), we get

$k.E. = \frac{3}{2}RT$ for 1 mole of the gas. .... (iv)

To get average kinetic energy per molecule, divide both sides of eqn. (iv) by the Avogadro's number, N i.e., the number of molecules present in one mole of the gas. Thus we have

$\text{Average K.E.} = \frac{3}{2}\frac{R}{N}T$

or $\overline{K.E.} = \frac{3}{2}kT$ .... (v)

Where $k = \frac{R}{N}$ is called Boltzmann constant.

From eqn. (v), we observe that

Average K.E. $\propto$ Absolute temperature.